Spring 4-1-2019

Does Teaching the History of Mathematics in High School Aid in Student Understanding?

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DOES TEACHING THE HISTORY OF MATHEMATICS IN HIGH SCHOOL AID IN STUDENT UNDERSTANDING?

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1 April 2019

Submitted in partial fulfillment of the requirements for

graduation with Honors

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Author Note:

Special thanks to Dr. Jeff Smith, academic advisor, for his guidance and support throughout this project, and to Danielle Jackson, cooperating teacher, for her guidance and willingness to let me conduct this research in her classroom.

Does Teaching the History of Mathematics in a High School Aid in Student Understanding?

Abstract

This research will study the effect teaching the history of mathematics in a high school classroom has on student understanding. To accomplish this, lessons both including and excluding historical background on different topics were taught in an Honors Algebra 2 class in the high school setting. This research aims to engage student learning and investigation of topics that normally do not draw a lot of student focus and spark a new or revived interest in mathematics for students by broadening lessons to include material of which students would not otherwise be exposed. The lessons themselves aim to engage other current and future educators to incorporate more historical and background information in their own lessons. The lessons allow students to think and learn as those who created and discovered the content being taught. In doing this, students will gain a deeper understanding of the material and a better ability to complete problems and the like in the lessons including historical topics.

Keywords: history of mathematics, secondary education, high school mathematics, mathematics education

Introduction

In this study, I will be investigating whether teaching historical background in tandem with mathematical concepts in a high school setting effects the understanding students have of the concepts introduced. According to James Crotty, many students have trouble fully grasping mathematical concepts in their high school classes. The Organization for Economic Cooperation and Development (OECD) stated that “the United States ranks significantly lower than average at mathematics comprehension, accounting for only 8.8 percent of all top-level mathematics performers at the K-12 level" (Crotty 2014). I am interested in finding out if teaching historical background to these particular topics will aid students in their understanding and comprehension of the material. While there are many different opinions and speedbumps surrounding incorporating mathematical history in the high school setting, many sources advocate for the integration of history into the classroom (Panagiotou 2014). Through this research, I plan to delve deeper into the effect this integration can have on the individual students as well as the class as a whole.

There are many different reasons from different sources about what purpose incorporating history into lessons serves in the high school setting. Fenaroli argues that integrating history into the mathematics classroom has two functions: "promoting the need of placing the development of mathematics in the scientific and technological context of a particular time and in the history of ideas and society," and "challenging one's perceptions of mathematics and mathematical concepts through making the familiar unfamiliar". Here, Fenaroli states that the reasoning for teaching history in mathematics should align with one of these functions, while I, along with Panagiotou, argue that it could also serve the purpose of helping students better understand difficult material. Mathematical history's inclusion in high school has many other reasons it is being advocated for, including its ability "to motivate interests, develop positive attitudes, unveil the nature of mathematics and mathematical activity, improve learning, and provide an image of the mathematical development in time and space, tightly linked to other science, culture and society" (Panagiotou 2014). Through these aspects of history, a deeper understanding and comprehension of the core concepts being taught will come.

Many school systems do not lend themselves to the insertion of more complex and historical topics into what instructors already have to teach to their students (Bussotti 2013). By incorporating some historical background into a few different topics rather than trying to infuse more information into every lesson, I will hopefully be able to bypass this issue. Although interest in mathematical history has grown in recent years, research regarding this topic is few and far between. This could be because of the "lack of didactical material, the gap between historians of mathematics and teachers of mathematics, and, especially, the lack of empirical research studies integrating history of mathematics education" (Panagiotou 2014). However, as a student, I have received information both on how to effectively teach mathematics to an adolescent audience and on mathematical history and philosophy. I hope to discover the effect this can have on student understanding, especially since there is not much research regarding this particular topic.

Based on this previous research and continued interest in the history behind mathematics and how it can influence student understanding, I hope to see a difference in the student's ability to solve problems, remember vocabulary, and how well they feel they are understanding material. Student learning and knowledge of mathematical history is also predicted to increase as a result. As current and future educators, we must be aware of what can be done to best help students fully grasp material. This carries great importance because effective teaching drives student learning and understanding, which can lead students to better career paths, better understanding of future educational topics, and better their drive to seek background information on other topics in which they are interested.

The class in which I am introducing these topics is Honors Algebra 2 at Whitehall-Yearling High School located in Whitehall, Ohio. Throughout my student teaching experience, I am teaching this course three times a day. In each class, I am teaching two lessons that include history topics that correspond to the content covered. To discover the effect of this, I am assessing students before and after learning the history topics and surveying them to gauge interest in the historical background. The lessons given follow the discoveries of Carl Friedrich Gauss, René Descartes, the Pythagoreans, Theodor Estermann, Euclid, and Isaac Newton, connecting to the concepts taught in the Algebra 2 curriculum. Both of these lessons also follow the problem-solving method of learning, connecting to DiNapoli's research on student perseverance in solving problems in the mathematics classroom. In challenging students to work through difficult problems, students are more engaged and better able to see connections (DiNapoli, 2018). This engagement leads students to be more active learners, allowing them to gain understanding of more difficult topics. Students in this study are also seeing applications of content in the history lessons taught. Here, student engagement is increased through "being asked interesting questions, solving novel problems, digging deeply into understanding a single topic, […] and discussing ideas about the subject with the teacher or students" (Jansen et al., 2018). Again, student engagement leads the student to have higher levels of interest and understanding.

Methodology

Materials

This study consists of five lessons, two of which were used as introductory lessons to give students an overview of two mathematicians who did work in the field of Algebra, and to get them thinking about the history behind the things they learn in class. The other three lessons are more interactive, discovery-based lessons in which students delve deeper into a particular concept from Algebra 2 and discover the more abstract thinking behind it, while also connecting the concept to a mathematician from history. The two introductory lessons included a short video, an activity, and an article with a corresponding fill-in-the-blank worksheet. The three discovery-based lessons connected more specifically to material. Each of these include a student-led discovery activity in which students work to learn more about a particular concept and incorporate the abstract thinking and mathematical history introduced in the two introductory lessons. Instructional materials for each can be found in Appendix A and formal lesson plans can be found in Appendix C.

Procedure

For the two introductory lessons, student learning was more structured, with direct instruction as the basis for each lesson. Here, students were asked to participate in a short activity, then work through an informational article and handout on their own or with their peers. Because these lessons dealt with introducing students to the more abstract style of thought and what mathematicians do, they were more conducive to direct instruction. The three discovery-based lessons, however, provide students with content that is best learned through self-led activities. This also allows them to use the abstract thinking skills that are introduced in the first two lessons. Each of these three discovery-based lessons include a self-paced activity tied to a concept in Algebra 2.

Lessons

Introductory Lessons

The first of the two introductory lessons taught students about Carl Friedrich Gauss, since he proved the Fundamental Theorem of Algebra, or that every polynomial has at least one root. Students in Algebra 2 learn various ways to solve a polynomial for its roots, so they use this theorem. Because this lesson is the students' first that exposure to the history of mathematics in this class, I started by asking them to rate from zero to ten how much they knew about the history of mathematics. The lesson itself consisted of having students add the sum of the numbers one through one hundred, because as a child, Gauss solved this problem in mere seconds. So, I set a timer for five minutes, and asked the students to add up the numbers one through one hundred. This, of course, resulted in some push back from the students, saying how hard the problem was and that they could not do it in only five minutes. After a few minutes passed, I gave them the hint that the numbers do not have to be added in order, and that instead, they could add them up in whatever order they wanted. A few students caught on, with one doing 1 + 99 and 25 + 75, and a couple adding pairs to ten. After the five minutes ended, I explained to them that we would be talking about Gauss and explained his connection to what they were learning. Then, I told them that he was able to find the sum in seconds as a child. I also told them the method that he used: finding fifty pairs that add to 101 and then multiplying 101 by fifty. Then, I showed a video explaining how Gauss found the sum, and generalized the method to find the sum of any amount of numbers with a closed formula. The video shown in class was *Gauss Summation* created by Dr. Jeffrey P. Smith on YouTube. An article was then provided for them, and they read through it, answering questions that went along with it. With this, they were learning more about Gauss' early life and what he contributed to mathematics, astronomy, and physics.

In general, the students responded well to both the change of pace in content and the subject of the lesson. Most had a hard time thinking about how to add the numbers one through one hundred, and for many it was the first time they had been introduced to the kind of abstract thinking done by mathematicians. For the most part, students do not learn abstract thinking in a high school math class, so this lesson helped ease them into that part of mathematics.

The second introductory lesson taught students about René Descartes during our unit on rational expressions and functions, because Descartes introduced modern notation for exponents, writing x^n rather than $x \cdot x \cdot x \cdot \ldots \cdot x$. For this lesson, I incorporated Descartes discovery of the Cartesian Coordinate Plane. Students were provided a blank coordinate plane, and the story of Descartes' discovery was explained, both by the video shown in class, *The Discovery of the Cartesian Coordinate System* created by Aqsa Asif on YouTube, and verbally by me after the video to ensure students are understanding how he went about labelling points on a grid. His discovery involved watching a fly on his tiled ceiling and wondering how to describe its location. He then counted from the lower-left corner how many spaces right and then up the fly had landed. To connect with this, students were prompted to plot a fly, or a point, somewhere on the provided coordinate plane and, using Descartes' method, describe the location of the fly. This was repeated a few times, with students drawing the fly in at least three locations. Students were then reminded that although this concept does not seem revolutionary now, at the time, no one had connected the concepts of functions and equations to a graphic representation. Students were then provided with an article, like that from the Gauss lesson, and a handout to go along with the article.

Because this article was not as challenging in content as the Gauss lesson, students were less responsive, but seemed to enjoy learning more about something they are used to seeing. Students responded well to the content and seemed more comfortable with the concept of learning about the history behind the mathematics they have already learned, having already participated in a math history lesson. Students also had less commentary about the topic being "too hard" as they had replied during the Gauss lesson.

Students were also given a survey asking similar questions to that of the first lesson at the end of this lesson. Students were asked again to rate their knowledge of math history on a scale from zero to ten, to list any mathematicians they knew, and then to rate on a scale from one to ten how much the math history made their understanding of the unit better, in their opinion. A copy of this survey is provided in Appendix B.

Discovery-Based Lessons

The discovery-based lessons work off what students learn in the introductory lessons. Here, students use their prior knowledge to work through more challenging content tied to the curriculum and mathematical history. Each of these lessons provides students with a particular topic from Algebra 2 and allows them to learn more about how it came to be, and who did important work with the concept.

The first of these lessons is a WebQuest that goes through different proofs of the irrationality of $\sqrt{2}$. The website students use was created through GoogleSites, and provides them with three different proofs, along with an article and questions about the mathematician tied to the proof. The first of the three proofs is from the Pythagoreans, the second using the Fundamental Theorem of Arithmetic, and the third using the proof by Estermann. Each proof corresponds with a mathematician, or in the case of the Pythagoreans, a school of mathematicians. During the lesson, students are prompted to choose one proof and explore both the corresponding mathematician and the proof itself. Students are also instructed to rewrite the proof in the form of a two-column proof, which is how proof is first introduced in Geometry in the high school curriculum.

The second of these lessons is a Desmos online graphing lab where students explore Descartes' Rule of Signs to determine the number of real zeros a polynomial function has. The lab begins with students finding unique coefficients for a fifth-degree polynomial. Students are then guided through Descartes' Rule of Signs, where they determine the number of sign changes for the positive and negative versions of their unique function. Students are also given a worksheet to go along with the activity, so they can write down each step, and keep track of their function as they go. Information about Descartes himself is not given during this lesson, as students received information about him during the second introductory lesson. Some background information about the Rule of Signs is given during the introduction to the activity.

The third discovery-based lesson is an activity going through Newton's Method for finding the zeros of a polynomial. This lesson takes students through Newton's Method of choosing a value close to the root of a function, creating a tangent line to that point on a graph, and finding where the tangent line crosses the x-axis to approximate the roots of a function. Here, students are introduced to Newton's Method with direct instruction, aided by though-provoking questions and student response. Students then work through a similar problem on their own. Because Newton is a well-known mathematician, the information on Newton himself is given informally at the beginning of the lesson.

Results

Data Analysis

The initial hypothesis stated that students would have an increase in knowledge of mathematical history, because lessons taught in the classroom have background in mathematical history, and students are therefore gaining exposure to the history of mathematics. To test this hypothesis, students were surveyed on both their ability to name mathematicians and their opinion of their mathematical history knowledge. The students were asked to list any mathematicians they knew on the survey, and the average amount per student doubled from the pre-survey to the post-survey. They were also asked to rate their perceived knowledge of math history on a scale of zero to ten. Again, the average here doubles from the pre-survey to the post-survey, although the average is on the lower end of the scale.

Table 1

Student Response to Pre-Survey and Post-Survey

Averages of Questions Asked on Survey (Rating on scale of 0 – 10)

Survey	Number of Mathematicians Named	Knowledge Rating
Pre-Survey	0.615	1.585
Post-Survey	1.24	3.82

Students were also asked to rate their opinion of the history lessons' effect on their understanding. Here, students were asked "on a scale of one to ten, how much do you think learning about mathematicians and math history helped you understand the topics we are learning in class?" The average rating here was 5.10, with a standard deviation of 2.34.

Table 2

Student Value Added to Unit

Student Ratings of History's Effect on Understanding (Rating on scale of 0 – 10)

Rating	Number of Students	Percent
1	2	5.12%
2	5	12.82%
3	3	7.69%
4	6	15.38%
5	6	15.38%
6	6	15.38%
7	5	12.82%
8	3	7.69%
9	1	2.56%
10	2	5.12%

It was also hypothesized that student understanding would improve in lessons including math history. Table 3 shows results of a paired t-test performed to compare student scores between a unit with no history topics included and student scores on units including Introductory Lessons 1 and 2.

Table 3

T-Test of Student Test Scores Between History and Non-History Lessons

Student summative scores between lesson without history and Introductory Lessons

Unit	t	df	p-value	Confidence Interval, 95%
Introductory Lesson 1	-2.8612	44	0.006436	[-11.5141, -1.997]
Introductory Lesson 2	3.2529	44	0.002197	[3.3141, 14.1081]

Here, a paired t-test was run to compare summative assessment scores from both units including Introductory History Lessons to those of the unit without any history topics included. This test yields p-values below 1%, indicating that the differences seen between scores are not occurring by chance. In the study, student assessment scores for the unit without history had an average of 81.4%. The assessment for the unit including Introductory History Lessons 1 and 2 had an average score of 87.65% and 70.8%, respectively. Because the t-scores for both are relatively high, it can be assumed that an effect is seen on student assessment scores. Here, although we see an effect on student assessment scores, this cannot specifically be attributed to the history lessons themselves. This change in scores between units, while it could be from the history lessons that are included, could also be because of differing levels of difficulty in content, the types of lessons in general included in the units, and general student understanding. The best way to see the effect the history lessons have on student assessment scores would be to compare units with similar levels of difficulty – with and without history lessons included. This effect would also be best seen if units with and without history lessons included similarly structured lessons as well, with each consisting of discovery-based lessons.

While it would be useful to include history lessons for one class and not include history lessons for another to study the differences, this creates an ethical issue in the field of education, where one class would be receiving more in-depth, rich lessons, while the other would only be getting the content. It also does not account for classes with varied levels of understanding across classes. Where one class may have a good understanding of content, another might not.

Limitations

Because this study took place over the course of a nine-week quarter in a high school math classroom, the full effects of incorporating mathematical history topics into the existing curriculum cannot be seen. This research would be seen in its full effect if conducted over the course of a semester or full year at the high school level, incorporating history lessons throughout different units across the curriculum. Because of the different content assessed in this study, the effect on student understanding on assessment scores is also not apparent. This study also only focused on Honors Algebra 2, so the general effect on student understanding in a different math class at the high school level is not seen. The number of historical topics in Algebra as opposed to Geometry or Pre-Calculus differ and connections between the history and curriculum can be more easily seen in different areas of study.

This study only included the two Introductory Lessons, again, because time constraints did not allow for each of the five lessons provided to be taught in full to students. The Discovery-Based Lessons are provided here are not included in data analysis, because they were not taught to students in the study. If the study were to take course over a school year, not only would all five history lessons provided here be able to be incorporated, there would be more opportunity to explore history topics for more concepts in the Algebra 2 curriculum.

Conclusions

Added Value in Understanding

One of the goals of this study was to see if any value is added to student learning and understanding by teaching mathematical history in a high school classroom. Based on the data from the surveys provided to students, students saw an increase in their own understanding by learning more about math history. During class, students exemplified similar levels of understanding based on informal formative assessments. Given instruction in the classroom, students in both lessons showed similar levels of conceptual understanding whether a history lesson was taught during the unit or not. This would be better seen in a longer study, so to further examine how history lessons effect student understanding on summative assessment, history lessons should be taught over a longer period of time. Students also were asked to rate their perceived understanding increase from learning about the history of mathematics. Here, the median average was five, so most students rated themselves at a five on a scale of one to ten regarding how well they thought the history lessons helped their understanding. This shows that students can connect both the history content and the unit, giving them a more well-rounded view of the topics covered throughout the unit and mathematics in general.

Student Knowledge of the History of Mathematics

The study also aimed to increase students' knowledge of math history by providing lessons and activities connected to new historical topics. In doing this, students are exposed to new mathematicians and the mathematical thought process. Here, students showed an increase in knowledge, but the average rating is still on the lower end of the scale. Given more history lessons throughout instruction during the year, student knowledge of math history would see a similar increase. Here, we see a doubling in student perceived knowledge, as well as a doubling in the knowledge of mathematicians. In this study, four students rated themselves lower on the post-survey than on the pre-survey, however, all four had an increase or no change in the number of mathematicians named. This leads to the assumption that students' perception on the history of mathematics in general has changed over the course of the study, rather than their knowledge on the subject. With this, it can be predicted that students going in to the study assumed their knowledge to be higher, but after learning more about two mathematicians that they had no prior knowledge of, their perception of their knowledge of math history decreases because their scope has widened, realizing the broad history that mathematics has. If this is the case, another goal of this study was to see students gain appreciation for the history of mathematics, which can be seen here.

Throughout the study, students were noticeably more engaged in the classroom during history lessons, asking questions and interacting with their peers, myself, and the material. If incorporated to the general curriculum of the class for the school year, one can expect student learning to improve. In both classes used in this study, students typically do not use the critical thinking skills introduced in the history lessons. Even though the classes are Honors-level classes, students do not typically think in an abstract sense, and through these history lessons, they are introduced to the notion of thinking of thing abstractly and the way mathematicians think. The lessons here introduce students to the way the world around us is constantly described by mathematics and how mathematicians discover new concepts by questioning the world around them.

Student knowledge was observed to increase, especially in the field of mathematical history, both in the context of mathematical history itself and abstraction. Student engagement is also observed to increase, which allows students to connect more with material, making it resonate with them more strongly, giving them a better understanding of content. Most importantly, student perceived understanding is increased through the incorporation of math history in the high school classroom. This gives students a higher confidence level with the content and processes in the curriculum and inspires them to think more critically and ask questions, whether it be in response to learning a concept, working through a problem, or questioning the world around them. After the history lessons, students were observed to ask more questions during both direct instruction and independent or small group work. This could be due to an increased sense of how mathematicians think and solve problems – by asking questions.

The implications of this study show that teaching mathematical history alongside the existing mathematics curriculum gives students a higher level of engagement and widens their scope of the world of mathematics. An increased level of student engagement allows students to further connect the content to the world around them, which gives them both a higher level of understanding and a higher confidence level with the content. A widened scope of the world of mathematics gives students a newfound appreciation for how the concepts they are learning in the classroom come to be and gives them a richer background for the content. This connects students to the field of mathematics and allows them to view content and the subject of mathematics in a new way. In general, adding history to the mathematics classroom has a positive effect on classroom environment and student learning, giving students a new way to view the subject of mathematics.

Future Work

Because of the effect seen on both student understanding and engagement during units with history lessons included, I plan on continuing to teach math history alongside the existing mathematics curriculum for high school. In doing this, the full effect of including history would be seen over the course of a school year, units would be more comparable in difficulty, and student understanding would be better assessed to compare between units with and without history lessons. When I am teaching in my own classroom, I will be able to fully incorporate math history from the beginning of the school year, giving students more time to learn more about the field of math history as well as the process of mathematical thought and conjecture. In doing this, I will continue to use my existing lessons, and plan additional lessons to connect to more concepts throughout the curriculum. This will not only allow me to see and study the full effect of the incorporation of math history in a high school classroom, but will also allow me to infuse my own interests in math history into my teaching practice, and hopefully spark interest in students in the field of mathematics and math history.

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Appendix A

Instructional Materials

Carl Friedrich Gauss

1777 – 1855

Carl Friedrich Gauss (1777-1855) is considered to be the greatest German mathematician of the nineteenth century. His discoveries and writings influenced and left a lasting mark in the areas of number theory, astronomy, geodesy, and physics, particularly the study of electromagnetism.

Gauss was born in Brunswick, Germany, on April 30, 1777, to poor, working-class parents. His father labored as a gardner and brick-layer and was regarded as an upright, honest man. However, he was a harsh parent who discouraged his young son from attending school, with expectations that he would follow one of the family trades. Luckily, Gauss' mother and uncle, Friedrich, recognized Carl's genius early on and knew that he must develop this gifted intelligence with education.

While in arithmetic class, at the age of ten, Gauss exhibited his skills as a math prodigy when the stern schoolmaster gave the following assignment: "Write down all the whole numbers from 1 to 100 and add up their sum." When each student finished, he was to bring his slate forward and place it on the schoolmaster's desk, one on top of the other. The teacher expected the beginner's class to take a good while to finish this exercise. But in a few seconds, to his teacher's surprise, Carl proceeded to the front of the room and placed his slate on the desk. Much later the other students handed in their slates.

At the end of the classtime, the results were examined, with most of them wrong. But when the schoolmaster looked at Carl's slate, he was astounded to see only one number: 5,050. Carl then had to explain to his teacher that he found the result because he could see that, 1+100=101, 2+99=101, 3+98=101, so that he could find 50 pairs of numbers that each add up to 101. Thus, 50 times 101 will equal 5,050.

At the age of fourteen, Gauss was able to continue his education with the help of Carl Wilhelm Ferdinand, Duke of Brunswick. After meeting Gauss, the Duke was so impressed by the gifted student with the photographic memory that he pledged his financial support to help him continue his studies at Caroline College. At the end of his college years, Gauss made a tremendous discovery that, up to this time, mathematicians had believed was impossible. He found that a regular polygon with 17 sides could be drawn using just a compass and straight edge. Gauss was so happy about and proud of his discovery that he gave up his intention to study languages and turned to mathematics.

Duke Ferdinand continued to financially support his young friend as Gauss pursued his studies at the University of Gottingen. While there he submitted a proof that every algebraic equation has at least one root or solution. This theorem had challenged mathematicians for centuries and is called “the fundamental theorem of algebra”.

Gauss’ next discovery was in a totally different area of mathematics. In 1801, astronomers had discovered what they thought was a planet, which they named Ceres. They eventually lost sight of Ceres but their observations were communicated to Gauss. He then calculated its exact position, so that it was easily rediscovered. He also worked on a new method for determining the orbits of new asteroids. Eventually these discoveries led to Gauss’ appointment as professor of mathematics and director of the observatory at Gottingen, where he remained in his official position until his death on February 23, 1855.

Carl Friedrich Gauss, though he devoted his life to mathematics, kept his ideas, problems, and solutions in private diaries. He refused to publish theories that were not finished and perfect. Still, he is considered, along with Archimedes and Newton, to be one of the three greatest mathematicians who ever lived.

Discovering Gauss

Carl Friedrich Gauss was a mathematician who lived from

____________________ – __________________

When he was young, Gauss calculated the sum of numbers from 1 – 100 within seconds, stunning his teacher. How did he do it?

$$1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100$$

What else did he do?

- Proved the Fundamental Theorem of Algebra, or that ______________________ ____ ____
- Found that a regular polygon with 17 sides could be drawn using just a ____________________ and __________________________
- Calculated the exact location of a dwarf planet named _________________ and worked on a new method to determine the _________________ of new asteroids
- Who are considered to be the 3 greatest mathematicians that ever lived?
    1. Gauss
    2.
    3.

René Descartes

1596 – 1650

René Descartes (31 March 1596 – 11 February 1650) was a French philosopher, mathematician, and scientist. A native of France, he spent about 20 years (1629–1649) of his life in the Dutch Republic after serving for a while in the Dutch States Army. He is generally considered one of the most notable intellectual figures of the Dutch Golden Age.

Descartes' influence in mathematics is apparent, since the Cartesian coordinate system was named after him. He is credited as the father of analytical geometry, the bridge between algebra and geometry, used in the discovery of calculus and analysis. Descartes was also one of the key figures in the Scientific Revolution.

Throughout the history of mathematics until Descartes, there was always a divide between algebra and geometry. On the one hand, we had the symbolic and abstract manipulation of numbers and unknown quantities, and on the other hand, we had the investigation of shapes and solids.

Descartes' analytical geometry unified these two fields. He pioneered the idea of representing algebraic forms and equations using geometric lines and curves on a coordinate plane. His basic ideas are still taught in high-school mathematics today, with students learning how to graph an equation like $y = 3x + 5$ as a line, or an equation like $y = x^4 - 7x + 12$ as a parabola. Descartes was also one of the first mathematicians to represent a variable multiplied by itself as the variable raised to a certain power, like

This combination of geometry and algebra was a significant precursor to the later development of calculus, and is such a central idea of modern mathematics that we take it for granted. Descartes' work was so fundamental that we refer to the coordinate system he invented as the "Cartesian plane."

Discovering Descartes

René Descartes was a mathematician from ________________
who lived from

___________________ – ______________________

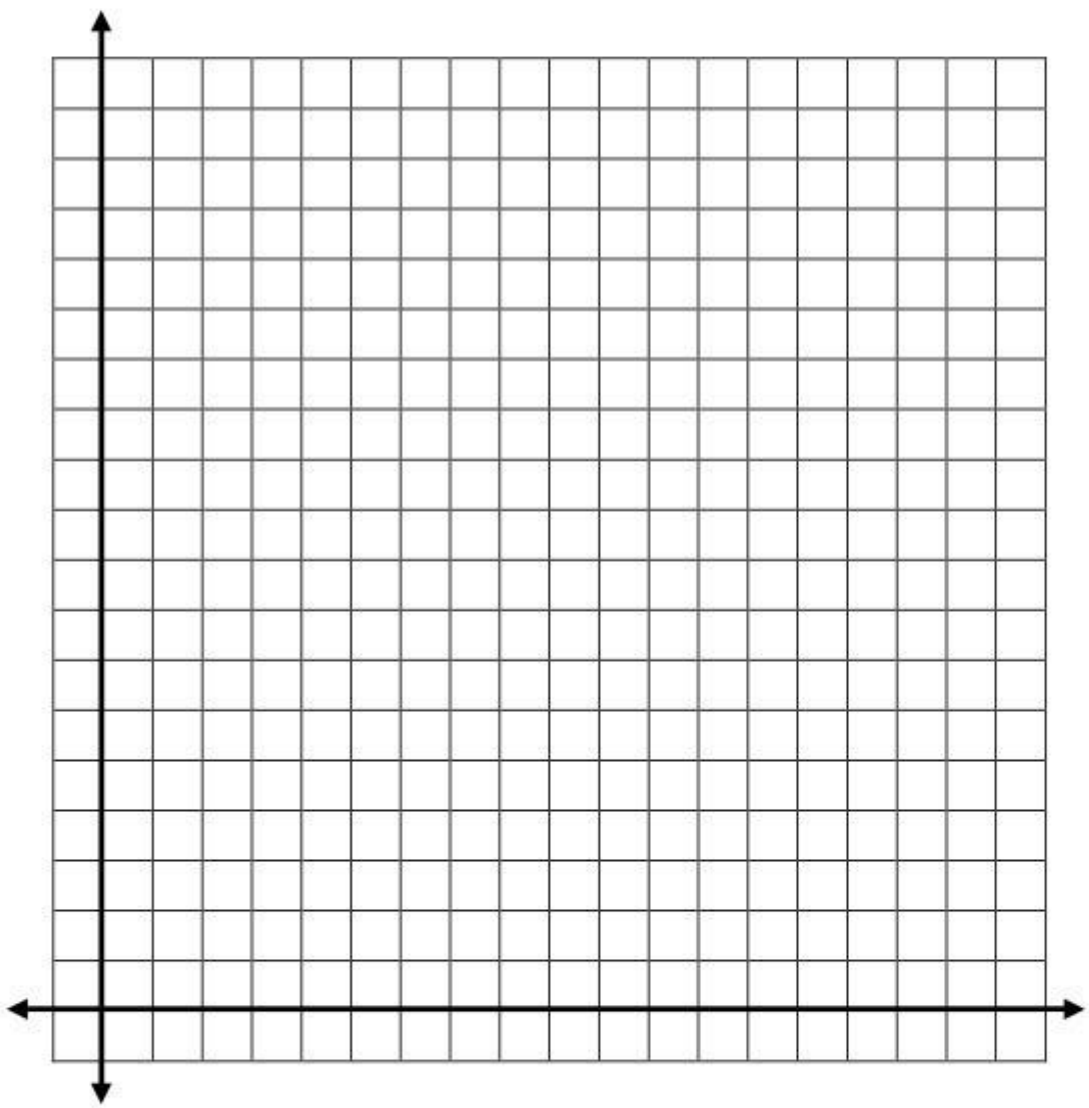
What did he contribute to mathematics?

- Known as the father of ____
- Created the Cartesian __________________________________, which we now use to graph functions.
- Was the first to write a variable multiplied by itself as the variable raised to _____________________________, like ____________________

How did he come up with the Cartesian Coordinate System?

Descartes' Coordinate System

When he was a child, René Descartes was ill for a long time, and was restricted to his bed. He noticed a fly on his bedroom ceiling, and wondered how he could describe the location of the fly on the square ceiling tiles.



Why is $\sqrt{2}$ Irrational?

Webquest Activity

Go to https://sites.google.com/otterbein.edu/sqrt2irrational/home

What is an **irrational number**?

Choose a proof, click on the button to take you to its page

Read through the information about the mathematician, fill out the form, then read through the proof.

After reading through the proof, complete the 2-column proof (on the back) with each step and explanation.

Which proof did you choose?

Who proved it?

Step	Explanation

Descartes' Rule of Signs – Desmos Lab

Go to student.desmos.com and type in the class code: CZFR9X

Slide 1: Introduction – What is Descartes' Rule of Signs

1. What is Descartes' Rule of Signs used for?

Slide 2: Find Different Values for Your Coefficients

2. Write down your function with your unique coefficients:

Slide 3: Positive x-intercepts

3. Go back to your graph from Slide 2. How many times does your graph cross the x-axis at a positive value?

Slide 4: Negative x-intercepts

4. Go back to your graph from Slide 2. How many times does your graph cross the x-axis at a negative value?

Slide 5: Find Sign Changes in Coefficients

5. Write down just your coefficients. How many times do they change from negative to positive or from negative to positive?

Slide 6: Sign Changes & Positive x-intercepts

6. Look at the number of sign changes you have.
    a. Your graph has at least ______ positive x-intercepts.
    b. Is your number of sign changes even or odd? (the number of positive x-intercepts must also be even/odd)
    c. Which numbers are less than your answer for (a) and even/odd?

Slide 7: Find Negative Sign Changes

7. Plug in your coefficients from #2.

Slide 8: Find Sign Changes of Negative Function

8. For even-powered exponents, keep your coefficients with the same sing. For odd-powered exponents, change your coefficients from negative to positive, or positive to negative.

   What are your new coefficients?

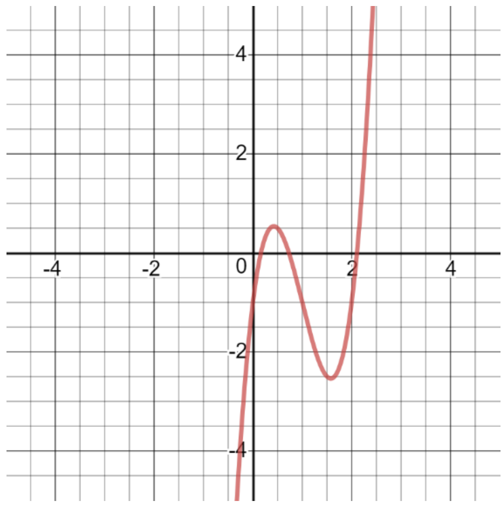
Slide 9:

9. Count your sign changes for your new coefficients from #8 – how many sign changes do you have?
   a. Your graph has at least ______ positive x-intercepts.
   b. Is your number of sign changes even or odd? (the number of positive x-intercepts must also be even/odd)
   c. Which numbers are less than your answer for (a) and even/odd?

Introducing… Newton's Method

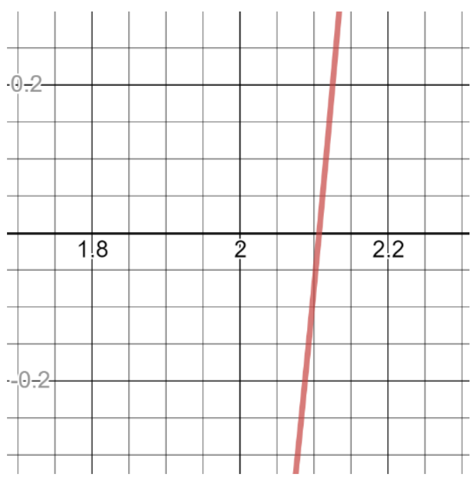
1. Use a calculator to write out as many digits as possible of $\sqrt{2}$. How many possible decimals are there?

Shown below is the graph of the function $f(x) = 4x^3 - 12x^2 + 8x - 1$.



How in the world would you find out where $f(x) = 0$, accurate to several decimal places?

As you can see, $f(x) = 0$ has a solution near $x = 2$. If we zoom in very closely on the graph around $x = 2$, we get the picture below:



2. Sketch the tangent line to $f(x)$ at the point (2, -1).
   a. What is the equation for this tangent line?

b. Where would this tangent line intersect the x-axis?

c. Why might you think that $f(x)$ crosses the x-axis near the place this tangent line does?

d. After seeing the tangent line graphed on the computer, would you say that $f(x)$ does cross the x-axis near where the tangent line crosses?

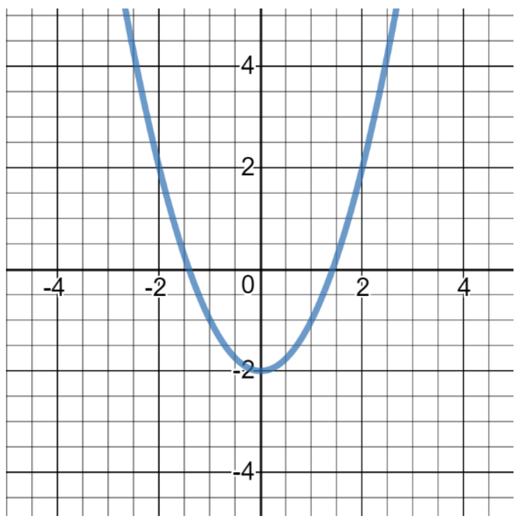
Key Idea: Newton's Method is the process of starting with a point near where $\boldsymbol{f(x) = 0}$ and using tangent lines to come up with (hopefully) better and better approximations for the root of $\boldsymbol{f(x)}$.

3. In the table below, record the numbers produced as we apply Newton's Method to the function $4x^3 - 12x^2 + 8x - 1 = 0$.

x_0	2
x_1	
x_2	
x_3	
x_4	

4. Use a calculator to find $f(x_4)$. Is it 0? How could you find an even more exact answer for $f(x) = 0$?

Now approximate solutions for a different equation. Below is the graph of $f(x) = x^2 - 2$



5. Use Newton's Method to approximate a solution for $x^2 - 2 = 0$, accurate to at least 5 decimal places. Record your approximations in the table below:

x_0		x_4		x_8	
x_1		x_5		x_9	
x_2		x_6		x_{10}	
x_3		x_7		x_{11}	

6. How does your value for x_{11} compare to the value of $\sqrt{2}$ from our warm-up?

7. How does our choice for x_0 affect what happens when we apply Newton's Method?

8. In the problem above, what happens if we choose $x_0 = 0$?

9. In general, when we want to use Newton's Method to approximate a solution to $f(x) = 0$, what are some good suggestions for x_0?

Appendix B

Surveys

History of Math Pre-Survey

1. On a scale of 1 – 10 how much do you think you know about the history of math? (circle one)

   0 1 2 3 4 5 6 7 8 9 10

2. List any famous mathematicians you know

History of Math Post-Survey

1. On a scale of 0 – 10 how much do you think you know about the history of math? (circle one)

   0 1 2 3 4 5 6 7 8 9 10

2. List any famous mathematicians you know

3. On a scale of 1 – 10 how much do you think learning about mathematicians and math history helped you understand the topics we are learning in class

   1 2 3 4 5 6 7 8 9 10

Appendix C

Formal Lesson Plans

Introductory Lesson 1 – Carl Friedrich Gauss

	Basic Information
Grade Level: 9-11 Subject: Honors Algebra 2	Length of Class: 46 min
	Big Ideas
Central Focus of the Unit *Describe the ENDURING UNDERSTANDING for the UNIT*	Students will have an understanding of polynomial functions, their graphs, and how to solve them for x. Students will also build upon their prior knowledge of factoring to solve functions that can be factored for x, and using previously learned and new methods to solve when we cannot factor. They will understand the meaning of the zero of a graph, be able to find zeros of a polynomial function using different methods, such as factoring, the quadratic formula, rational/irrational root theorem, and graphing.
Today's Essential Question *Write a thought-provoking question to drive today's lesson.*	What is the Fundamental Theorem of Algebra, who proved it, and how do mathematicians think? In what ways can thinking more abstractly and generally impact how we see and do mathematics?
	Specific Standards and Objectives
CCSS Standards in Math *Identify by CCSS Code.No and copy the text.*	CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively CCSS.MATH.PRACTICE.MP7 Look for and make use of structure CCSS.MATH.CONTENT.HSA.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial
Math Process Standard *Identify the NCTM Process Standard being targeted.*	Connections Problem Solving Reasoning
Student Learning Objectives *Describe the knowledge, skills, and dispositions*	• Students will gain an understanding of how mathematicians think and solve problems (Knowledge)

to be attained by the end of today's lesson. These should be measurable and should be assessed. The student will …	• Students will gain an appreciation for the history of mathematics and how mathematicians contributed to the subject (Dispositions) • Students will see and appreciate the connection between content and its discovery (Dispositions)
	Assessments and Evaluations
Tools for Assessment *Detail methods for monitoring understanding and determining attainment of SLO's. (Label according to Pre-, Formative, or Summative)*	Pre-assessment: • Students complete the pre-survey, rating their knowledge of math history and listing any mathematicians they know Formative: • Students complete the Gauss Fill-In Worksheet and turn in.
Evaluation *Articulate the criteria you will use to determine how well students are understanding or attaining the SLO's. (For example: grading scheme, rubrics)*	Students are informally evaluated throughout the lesson to assess both attitude towards math history and level of understanding connections between content and its history. Students are also given a completion grade for the Gauss Fill-In worksheet, to ensure that students complete the handout, giving them a deeper understanding of Gauss' life and contributions to mathematics. Scores are given out of 3 points, with 3 being completed fully, 2 being partial completion (more than half), 1 being partial completion (less than half), and 0 being incomplete.
	Lesson Outline
Anticipatory Set *Describe the launch/hook for today's lesson.*	Students are asked at the beginning of class to rate their knowledge of the history of mathematics on a scale of 1 – 10 and then list any famous mathematicians they know. They are then asked to add up the sum of the numbers 1 – 100, without a calculator.
Sequence of Activities *List (in order) the key learning experiences/tasks that will help students to attain the SLO's. Approximate the amount of time for each experience/task.*	1. Warm-up/bellwork is displayed on the board as students come in to class, give first 5 minutes of class for students to work on it, then go over together as a class, asking questions to assess students' prior knowledge and understanding of previous lesson (10 min) 2. Inform students that today's lesson will be talking about Carl Friedrich Gauss, the mathematician who proved the Fundamental Theorem of Algebra, which is what we have been using to find the zeros of polynomial functions. Introduce the fact that when he was a child, he was asked by his teacher to add up the numbers from 1 – 100, and was

	able to do it in seconds. Tell students they will have 10 minutes to add up the numbers 1 – 100, and they are not allowed to use a calculator. After 4-5 minutes have passed, give them the hint that the numbers do not have to be added in order. (13 min) 3. After timer is done, explain to students how Gauss was able to find the sum so quickly: he noticed that the first and last numbers have a sum of 101, the second and second-to-last numbers have a sum of 101, and so on. So, there are 50 pairs of numbers that add to 101, making the total sum 101 x 50 = 5050. (3 min) 4. Play video on Gauss and adding up a sum of any amount of numbers, 1 – n (5:42) 5. Pass out Gauss article and fill-in worksheet, letting students know to read the article to fill in the blanks on the worksheet. Remind students why we are learning more about Gauss and how he connects to what we are learning in class. (1 min) 6. Students work on worksheet for the remainder of the period, circle the room to answer any student questions and make sure students are on task (remainder of period).
Closure *Describe the closing of today's lesson (relating back to the essential question).*	Students are reminded how Gauss connects back to the content they are learning in the unit, and how mathematicians think through problems. Students are also made aware of more abstract reasoning, and formulaic thinking, so the formula found in the video to add up a sum of numbers is connected back to the original problem, 1 + 2 + … + 100.
	Meeting Needs
Engaging All Learners *Describe your strategy for providing differentiation in today's lesson. (Cultural/developmental responsiveness)*	Complicated terms in the article provided for students are easily defined in the article to assist in understanding for those with any language barriers. Students are able to work independently or with peers to enable student participation and understanding. Information on Gauss' work to find the sum is provided both verbally in class and in the video shown, with additional explanation where needed.
Resources and Materials *List specific resources and materials required for today's lesson.*	Pre-knowledge survey Blank paper Gauss Summation video: https://www.youtube.com/watch?v=k9MmrXypbUY&t=196s Gauss Article Gauss Fill-In Worksheet

Proactive Planning
What will challenge the success of this lesson? How can you circumvent problems? How will you navigate difficulties? Challenge #1: Students may not see connection between history and content Navigating challenge #1: Students are continuously reminded throughout instruction and explanations how the history connects back to rational functions – Gauss proved the Rational Root Theorem, which we have used to solve a polynomial for its roots when we cannot factor.

Introductory Lesson 2 – Descartes

Basic Information	
Grade Level: 9-11	Subject: Honors Algebra 2 Length of Class: 46 min
Big Ideas	
Central Focus of the Unit *Describe the ENDURING UNDERSTANDING for the UNIT*	Students will have an understanding of rational expressions and functions, knowing how to simplify and perform operations on a rational expression and find the zeros of a rational function to build upon their knowledge from previous units about finding zeros of a polynomial function. They will understand what the zero of a graph is, and what those look like for a rational function.
Today's Essential Question *Write a thought-provoking question to drive today's lesson.*	Who is René Descartes, and how does he connect to rational expressions? What did he discover in the field of mathematics? How is Descartes similar or different to Gauss? What types of things do mathematicians do when thinking about the world?
Specific Standards and Objectives	
CCSS Standards in Math *Identify by CCSS Code.No and copy the text.*	CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively CCSS.MATH.PRACTICE.MP7 Look for and make use of structure CCSS.MATH.CONTENT.HSA.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
Math Process Standard *Identify the NCTM Process Standard being targeted.*	Connections Problem Solving Reasoning

Student Learning Objectives *Describe the knowledge, skills, and dispositions to be attained by the end of today's lesson.* *These should be measurable and should be assessed.* *The student will …*	• Students will gain an understanding of how mathematicians think and solve problems (Knowledge) • Students will gain an appreciation for the history of mathematics and how mathematicians contributed to the subject (Dispositions) • Students will see and appreciate the connection between content and its discovery (Dispositions)
	Assessments and Evaluations
Tools for Assessment *Detail methods for monitoring understanding and determining attainment of SLO's.* *(Label according to Pre-, Formative, or Summative)*	Formative: • Students complete the Descartes Fill-In Worksheet Summative: • Students complete the Post-Survey, rating their knowledge of math history, mathematicians, and how much learning about math history effects their understanding.
Evaluation *Articulate the criteria you will use to determine how well students are understanding or attaining the SLO's.* *(For example: grading scheme, rubrics)*	Students are informally evaluated throughout the lesson to assess both attitude towards math history and level of understanding connections between content and its history. Students are also given a completion grade for the Gauss Fill-In worksheet, to ensure that students complete the handout, giving them a deeper understanding of Gauss' life and contributions to mathematics. Scores are given out of 3 points, with 3 being completed fully, 2 being partial completion (more than half), 1 being partial completion (less than half), and 0 being incomplete.
	Lesson Outline
Anticipatory Set *Describe the launch/hook for today's lesson.*	Students are show a video about Descartes, describing how he thought to label the location of a fly on his tiled ceiling. This introduces students to the thought process he had and what he discovered
Sequence of Activities *List (in order) the key learning experiences/tasks that will help students to attain the SLO's.*	1. Students complete the warm-up related to the unit on the board as they come in. They work independently for the first 5 minutes of class, then go over as a class (~10 min) 2. Inform students that we will be doing another history lesson, this one over René Descartes, who connected the fields of algebra and geometry. This connects to what we're currently doing because he also came up with the notation for exponents, which is what we used to simplify

Approximate the amount of time for each experience/task.	rationals. Also let students know that we will be graphing rational functions in a couple of weeks, so they will be learning more about where the concept of graphing comes from. (2 min) 3. Show students the video on Descartes, which highlights the thought process followed to create the Cartesian Coordinate Plane (1:20) 4. Reinforce to students how he thought to use a coordinate system to describe the location of a fly on his ceiling (by counting how many tiles horizontally and vertically from the corner) (1 min) 5. Pass out Descartes handout, letting students know to start on the side of the handout with the coordinate plane (1 min) 6. Ask students to draw a "fly" on their grid, then draw its other locations – flies don't just stay in one place. Students are then asked to plot points the way Descartes did, find its location by counting horizontally then vertically. (3 min) 7. Remind students that although it does not seem "revolutionary" or new to us, attaching a point with its coordinates changed the way we are able to see math, giving a visual connection to functions (2 min) 8. Pass out Descartes article, giving students instructions to read through it and fill in blanks on the opposite side of the handout (1 min) 9. Students work independently on handout and article, circle room to ensure students are on task and answer any questions students may have (remainder of class) 10. Pass out Post-Survey as an exit ticket, asking students to fill out and return before the end of the period (1 min)
Closure *Describe the closing of today's lesson (relating back to the essential question).*	Students are reminded that graphing functions was something revolutionary when Descartes made this connection between algebra and geometry, although it doesn't seem like it nowadays. Students are also reminded that we will be connecting the algebraic concept of rational functions to their graphs in the coming weeks, so we will be connecting the algebra and geometry concepts just like Descartes.
	Meeting Needs
Engaging All Learners *Describe your strategy for providing differentiation in today's lesson. (Cultural/developmental responsiveness)*	Complicated terms in the article provided for students are easily defined in the article to assist in understanding for those with any language barriers. Students are able to work independently or with peers to enable student participation and understanding. Information on Descartes is given both verbally in class, in writing in the article, and in the video.

Resources and Materials *List specific resources and materials required for today's lesson.*	Warm-up (GoogleDrive) Descartes Video: https://www.youtube.com/watch?v=dSKIN_EKJNA&t=3s Descartes Handout Descartes Article Post-Survey
Proactive Planning	
What will challenge the success of this lesson? How can you circumvent problems? How will you navigate difficulties? Challenge #1: Students may not see connection between history and content Navigating challenge #1: Students are continuously reminded throughout instruction and explanations how the history of the Cartestian Coordinate System connects back to rational functions – Descartes came up with exponential notation and connected functions to their graphs	

Discovery-Based Lesson 1 – Irrationality of $\sqrt{2}$

Note: Because these lessons were not fully incorporated into the classroom for this study, Central Focus for the Unit is not included in the formal lesson plan.

Basic Information		
Grade Level: 9-11	Subject: Honors Algebra 2	Length of Class: 46 min
Big Ideas		
Central Focus of the Unit *Describe the ENDURING UNDERSTANDING for the UNIT*		
Today's Essential Question *Write a thought-provoking question to drive today's lesson.*	In what ways can we prove that $\sqrt{2}$ is an irrational number? How do mathematicians think when trying to show something? How can we make sense of each step in a proof, while trying to get to final step?	
Specific Standards and Objectives		
CCSS Standards in Math *Identify by CCSS Code.No and copy the text.*	CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others	

<table>
<tr><td>Math Process Standard
Identify the NCTM Process Standard being targeted.</td><td>Connections
Problem Solving
Reasoning</td></tr>
<tr><td>Student Learning Objectives
Describe the knowledge, skills, and dispositions to be attained by the end of today’s lesson.
These should be measurable and should be assessed.
The student will …</td><td>• Students will gain an understanding of how mathematicians think and go about proving something (Knowledge)
• Students will learn more about how mathematicians lived and worked (Knowledge)
• Students will gain an appreciation for the different ways in which we can approach a problem (Dispositions)</td></tr>
<tr><td colspan="2" align="right">Assessments and Evaluations</td></tr>
<tr><td>Tools for Assessment
Detail methods for monitoring understanding and determining attainment of SLO’s.
(Label according to Pre-, Formative, or Summative)</td><td>Formative:
• Students are asked to complete a handout that outlines the proof they look through during the lesson and rewrite it in a two-column format, showing that they are working to understand each step and the reasoning behind it</td></tr>
<tr><td>Evaluation
Articulate the criteria you will use to determine how well students are understanding or attaining the SLO’s.
(For example: grading scheme, rubrics)</td><td>Student understanding is both informally assessed throughout the lesson through observation, and formally assessed with the handout students turn in at the end of the lesson.

Student understanding on the handout is graded for completion, but made note of – students are grouped based on high, medium, and low comprehension of proof.</td></tr>
<tr><td colspan="2" align="right">Lesson Outline</td></tr>
<tr><td>Anticipatory Set
Describe the launch/hook for today’s lesson.</td><td>Students are introduced to the irrationality of $\sqrt{2}$, by reminding them of different groups of numbers, and what is included.</td></tr>
<tr><td>Sequence of Activities
List (in order) the key learning</td><td>1. Warm up as students come in to class – students are asked to list numbers from each of the following groups of numbers: (5 min)</td></tr>
</table>

<table>
<tr><td>experiences/tasks that will help students to attain the SLO's. Approximate the amount of time for each experience/task.</td><td>Natural/Whole Numbers
Integers
Rationals
Irrationals
2. As students complete the warm-up, review together as a class, asking students to share responses for each group. Students are then asked to define what an irrational number is, and $\sqrt{2}$ being irrational is introduced (5 min)
3. Pass out $\sqrt{2}$ Irrationality handout and have students get a ChromeBook and log on to the provided website (2 min)
4. Inform students to choose one of the three proofs displayed on the home page, go through the article and questions provided, then read through and write the steps of the proof (2 min).
5. Students work through Webquest on their own. Circle the room to ensure students are working and to answer any questions they may have (remainder of period)
6. Ask students to think about how we might go about proving that a different number is irrational, say $\sqrt{5}$, or $\sqrt{7}$. This gets them thinking about how mathematicians would go about solving a problem and finding explanations for each step along the way. (2 min)</td></tr>
<tr><td>Closure
Describe the closing of today's lesson (relating back to the essential question).</td><td>Students are asked to think about how a mathematician would go about proving a different number is irrational, which encourages them to reflect upon how mathematicians think and problem solve. They are also encouraged to use this method of thinking and solving in the other concepts in class.</td></tr>
<tr><td colspan="2" align="right">Meeting Needs</td></tr>
<tr><td>Engaging All Learners
Describe your strategy for providing differentiation in today's lesson. (Cultural/developmental responsiveness)</td><td>Students work through proofs on their own, so their individual questions can be answered on a one-on-one basis and they can work at their own pace.

Student choice is also involved in this lesson, so students have different options and don't feel as limited during class.</td></tr>
<tr><td>Resources and Materials
List specific resources and materials required for today's lesson.</td><td>Warm-up (GoogleDrive)
WebQuest Handout
ChromeBooks</td></tr>
</table>

Proactive Planning
What will challenge the success of this lesson? How can you circumvent problems? How will you navigate difficulties? Challenge #1: Students may have a hard time working through and making sense of the proofs provided Navigating Challenge #1: Encouraging students to ask questions of both their peers and myself allows them to make sense of the parts of the proofs that aren't as clear to them right away. Proofs are also rewritten on the GoogleSite to be more conducive to high school students' proof knowledge.

Discovery-Based Lesson 2 – Descartes' Rule of Signs

Basic Information		
Grade Level: 9-11	Subject: Honors Algebra 2	Length of Class: 46 min

Big Ideas	
Central Focus of the Unit *Describe the ENDURING UNDERSTANDING for the UNIT*	
Today's Essential Question *Write a thought-provoking question to drive today's lesson.*	How can we use Descartes' Rule of Signs to find the zeros of a function? How do the coefficients of a function connect to its x-intercepts?
Specific Standards and Objectives	
CCSS Standards in Math *Identify by CCSS Code.No and copy the text.*	CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively CCSS.MATH.PRACTICE.MP4 Model with mathematics CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically
Math Process Standard *Identify the NCTM Process Standard being targeted.*	Connections Problem Solving Reasoning
Student Learning Objectives	• Students will be able to define and apply Descartes' Rule of Signs to find the number of x-intercepts of a polynomial function (Skills)

<table>
<tr><td>Describe the knowledge, skills, and dispositions to be attained by the end of today's lesson.
These should be measurable and should be assessed.
The student will …</td><td>Students will gain an understanding of Descartes' Rule of Signs and how it works for a polynomial function (Knowledge)
Students will connect history about Descartes from Introductory History Lesson to current lesson, and appreciate the connections between graphs and functions (Dispositions)</td></tr>
<tr><td colspan="2" align="right">Assessments and Evaluations</td></tr>
<tr><td>Tools for Assessment
Detail methods for monitoring understanding and determining attainment of SLO's.
(Label according to Pre-, Formative, or Summative)</td><td>Formative:
Students are informally assessed throughout lesson to gauge understanding of Rule of Signs and functions in general
Students are formally assessed on submitted responses on Desmos and handout turned in</td></tr>
<tr><td>Evaluation
Articulate the criteria you will use to determine how well students are understanding or attaining the SLO's.
(For example: grading scheme, rubrics)</td><td>Desmos lab is graded based on completion (student progress can be monitored online)
Handout is graded based on accuracy – student responses are given feedback and returned to students</td></tr>
<tr><td colspan="2" align="right">Lesson Outline</td></tr>
<tr><td>Anticipatory Set
Describe the launch/hook for today's lesson.</td><td>Students are asked to think back to Descartes from introductory lesson… what did he do? How did he connect geometry and algebra?</td></tr>
<tr><td>Sequence of Activities
List (in order) the key learning experiences/tasks that will help students to attain the SLO's.
Approximate the amount of time for each experience/task.</td><td>Warm-up on board when students come in to the classroom – students are asked to think back to Descartes (what did he do?) (5 min)
Students are asked to share out their responses with the class, and are asked how he connected geometry and algebra (5 min)
Students are informed about Descartes' Rule of Signs, and what it does – connects a polynomial's coefficients to its zeros (2 min)</td></tr>
</table>

	4. Pass out Descartes Rule of Signs Handout, and have students get out a ChromeBook and go to student.desmos.com and type in the class code at the top of the handout (1 min) 5. Students work independently on Desmos lab, and are reminded as they get started to write down work on their handout, and type in responses on Desmos (remainder of class) 6. Circle the room to both ensure students are completing their work and are staying on task and to answer any student questions 7. Students are reminded at the end of class to think about the connections between functions and their graphs, namely the x-intercepts.
Closure *Describe the closing of today's lesson (relating back to the essential question).*	Students are reminded at the end of class to think about the connections between functions and their graphs, namely the x-intercepts.
	Meeting Needs
Engaging All Learners *Describe your strategy for providing differentiation in today's lesson. (Cultural/developmental responsiveness)*	Students are able to work through activity at their own pace, and ask questions where needed. Students are also working independently, allowing for more one-on-one work with students who show lower understanding for the unit based on previous formative assessment.
Resources and Materials *List specific resources and materials required for today's lesson.*	Warm-up (GoogleDrive) Descartes Rule of Signs Handout ChromeBooks
	Proactive Planning
What will challenge the success of this lesson? How can you circumvent problems? How will you navigate difficulties? Challenge #1: Students may not remember major points from Descartes Introductory Lesson Navigating Challenge #1: During warm-up, ask students to either get out their Descartes article from the Introductory Lesson or to think back, so students have time before the lesson gets started to gather their thoughts on Descartes	

Discovery-Based Lesson 3 – Newton's Method

	Basic Information
Grade Level: 9-11 Subject: Honors Algebra 2	Length of Class: 46 min
	Big Ideas
Central Focus of the Unit *Describe the ENDURING UNDERSTANDING for the UNIT*	
Today's Essential Question *Write a thought-provoking question to drive today's lesson.*	How did Newton find approximate solutions to equations of the form $f(x) = 0$?
	Specific Standards and Objectives
CCSS Standards in Math *Identify by CCSS Code.No and copy the text.*	CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively CCSS.MATH.PRACTICE.MP7 Look for and make use of structure
Math Process Standard *Identify the NCTM Process Standard being targeted.*	Connections Problem Solving Reasoning
Student Learning Objectives *Describe the knowledge, skills, and dispositions to be attained by the end of today's lesson. These should be measurable and should be assessed. The student will …*	• Students will understand and be able to explain the idea behind Newton's Method (Knowledge) • Students will be able to find the recursive equation for finding x_{k+1} from x_k given the function $f(x)$ (Skills) • Students will understand how the solution approximated by Newton's Method depends on the initial guess, x_0. (Skills)
	Assessments and Evaluations
Tools for Assessment	Formative:

Detail methods for monitoring understanding and determining attainment of SLO's. *(Label according to Pre-, Formative, or Summative)*	• Students will complete a worksheet containing questions to be answered both during instruction and on their own as they practice Newton's Method in class
Evaluation *Articulate the criteria you will use to determine how well students are understanding or attaining the SLO's.* *(For example: grading scheme, rubrics)*	Students are graded based on completion, but feedback is provided on handout when turned in to direct student learning.
	Lesson Outline
Anticipatory Set *Describe the launch/hook for today's lesson.*	Students are asked to find a numerical answer to an equation without a whole number solution.
Sequence of Activities *List (in order) the key learning experiences/tasks that will help students to attain the SLO's. Approximate the amount of time for each experience/task.*	1. Warm-up on the board as students come in to the classroom – students are asked to use their calculators to give a numerical answer for the solution of $x^2 - 2 = 0$ (5 min) 2. Students are then asked to share out their solutions with the class, and then asked how they think their calculators can come up with the solution. Ask them how many decimal places their calculator provides, and how many decimal places there would be for the solution ($\sqrt{2}$). Explain that they will be learning the method for finding the approximate solution for an equation with a "messy" solution to as many decimal places as they want (5-10 min) 3. Pass out Newton's Method Handout, while explaining who Isaac Newton was, giving some background on his connection to math (1 min) 4. Present the equation $f(x) = 4x^3 - 12x^2 + 8x - 1$. Ask students to graph it on their calculators, and have a visual of the graph on the board as they do. Ask students to look and see which numbers the solutions are close to, and then focus on the solution near $x = 2$. (3 min) 5. Open the first Geogebra Applet, showing the same graph zoomed in near $x = 2$. Ask students to predict in which

	direction the function will intersect the x-axis. Then, advance the applet to draw the tangent line and ask the class what the equation of this line would be at $x = 2$, and where it would cross the x-axis. (5 min) 6. Advance the applet to the next step, making x_1 appear. Zoom out far enough to show the intersections of the x-axis with both the graph of the function and the tangent line. Ask students what they observe – did they make a good guess for the x-intercept? What do we actually get out when we plug in our predicted value? Ask again in which direction students think the zero will be. (5 min) 7. Zoom out again, showing x_2 and x_3. Point out that x_0, x_1, x_2, and x_3 get successively closer to where the function crosses the x-axis, and that x_2 and x_3 agree on the first 3 decimal places, allowing us to assume the solution will begin $x = 2.107$ ... (5 min) 8. Ask students again to find the solution for $x^2 - 2 = 0$, this time using Newton's Method to find an answer accurate to 5 decimal places. Let students know they can choose different values for x_0 – negative, positive, or 0. (remainder of class) 9. Circle the room to answer any student questions and make sure students are on task. 10. Let students know that we will go over their solutions together as a class the next day, using another Geogebra applet to see the three cases for x_0.
Closure *Describe the closing of today's lesson (relating back to the essential question).*	Students are asked to relate back to the function we looked at in the warm-up and connect it to Newton's Method to find an approximate solution.
	Meeting Needs
Engaging All Learners *Describe your strategy for providing differentiation in today's lesson. (Cultural/developmental responsiveness)*	Students are able to see visual representations of each step with the Geogebra applet, giving those with different learning modalities opportunity to see the reasoning behind the concepts in a new way.
Resources and Materials *List specific resources and materials required for today's lesson.*	Warm-up (GoogleDrive) Introducing… Newton's Method Handout Graphing calculators Geogebra Applets (and projector)

Proactive Planning
What will challenge the success of this lesson? How can you circumvent problems? How will you navigate difficulties? Challenge #1: Students may have trouble finding values for x_n on their own. Navigating Challenge #1: If students seem to be struggling during instruction with how we choose values for x_n, assign groups of students to choose positive, negative, and 0 for x_0, and go forward from there.