7-1994

An Integrated Infrastructure Management System

William V. Harper
Otterbein University, wharper@otterbein.edu

Kamran Majidzadeh

AN INTEGRATED INFRASTRUCTURE MANAGEMENT SYSTEM

William V. Harper
Director of Research
Resource International, Inc.

Kamran Majidzadeh

Westerville, Ohio, USA

Paper prepared for presentation at the
1994 International Road Federation Conference
Calgary, Alberta, Canada

ABSTRACT

This paper describes an integrated infrastructure system (with emphasis on pavement and bridge management systems) that allows cost minimization or benefit maximization. It integrates separate infrastructure systems so that management may optimally allocate scarce resources across the combined systems. Fuzzy set theory is used in these optimizations to better address the desirability or undesirability of the condition states used to categorize the infrastructure segments modeled. Both steady-state and multi-year models are part of this system. The full mathematical structure of the simpler steady-state problem is presented, and sample multi-year output showing the integrated budgets is given.

1. INTRODUCTION

An integrated infrastructure management system has been developed. A particular implementation of it called the Highway Maintenance Management System (HMMS) is the primary focus of this paper. The integrated infrastructure management system has the capability to link diverse management systems so that an overall optimal allocation of budgetary resources can be made across all the management systems linked together. This allows a better allocation of budget than the arbitrary subdivision of funds for each system. However, the agency can run this integrated system by specifying separate budgets for each system rather than letting the integrated system make a globally optimal decision. In either case, optimal budgets are developed within the various strata within each system. To make this abstract concept more clear, this paper describes the HMMS that integrates a Pavement Management System (PMS), a Bridges & Structures Management System (B&SMS), and a Non-Pavement Management System. A relational database (ORACLE) is used to perform the needed data storage and retrieval functions. This paper focuses on the integration of the PMS and B&SMS. The full integration with the Non-Pavement Management System may be found in References {1, 2, or 3}.

The HMMS is a flexible modular system that can be easily adapted to meet various needs. The particular adaptation presented here is for a given client but it can be modified easily for other applications. This integrated system allows the optimal allocation of the budget across the various subsystems, e.g., across the PMS and B&SMS in this paper. Thus it is not necessary to make an arbitrary division of the budget into the subsystem (PMS and B&SMS) - rather an optimal division can be determined by the HMMS.

The PMS and B&SMS steady-state and multi-year results may be optimized using either cost minimization or benefit maximization. The particular PMS and B&SMS descriptions used in the paper are for the Kingdom of Saudi Arabia and are easily modified to fit different agency needs. The PMS is divided into 9 strata based on 3 levels of climate and 3 functional classes. The condition state variables for the PMS are rutting (3 levels), cracking (3 levels), delta cracking - 1 year change in cracking (3 levels), roughness (3 levels), and index to first crack (4 levels). This results in 324 condition states. There are 17 possible maintenance actions with a feasible subset for each condition state. In the cost minimizations models (references {2,3}), management specifies desired performance levels and the optimization finds the lowest cost plan that will meet the performance goals. In the benefit maximization models, benefits based on fuzzy set memberships and importance weights are maximized subject to budgetary controls.

The B&SMS is divided into 43 strata with 36 for bridges, 6 for culverts, and 1 for tunnels. The 36 bridge strata result from 3 climates, 6 major bridge types, and 2 functional classes. Culverts are not subdivided by type in the optimizations and thus have only 3 climates and 2 functional classes resulting in 6 strata. The condition state variables depend on the stratum. As an example, steel bridges have deck (4 levels), superstructure (4 levels), substructure (4 levels), superstructure-age (3 levels), and substructure-age (3 levels) for a total of 576 condition states. For this bridge type there are 40 maintenance scopes (e.g., deck repair) with a selected subset feasible for each condition state. References {4,5} describe this in more detail.

The PMS and B&SMS are modular systems with prediction, cost, optimization, packaging, and comparator modules. The prediction modules determine the transition probabilities that estimate the degradation rates for the PMS or B&SMS segments. In the PMS a segment is a 1 km single lane of road. In the B&SMS, the definition of a segment depends on the stratum. For steel bridges it is a superstructure span with a substructure pier or abutment. The survey results are converted to condition states as described in reference {4} and are used in Bayesian updating algorithms to adapt the transition probabilities to the actual environmental conditions encountered. The cost module determines the action/scope optimization costs. This paper focuses on the optimization module. The packaging module takes the selected optimal stratum solutions and makes assignments to the actual segments. The optimization selections are made more specific and detailed cost estimates are created in a variety of different formats to satisfy management needs. The comparator module provides feedback on the system performance and implementation.

2. FUZZY SET THEORY ADAPTATIONS

In classical set theory, each "object" (e.g., condition state) is either a member of a set or it is not. As fractals have stretched the boundaries of numerous disciplines to consider non-integer dimensions to supplement the integer dimensions found in classical science, fuzzy set theory expands the concept of the membership of an object in a set to be any value on the continuum [0,1.0] with larger values representing a higher or stronger degree of membership in the set. Classical sets are special cases of fuzzy sets in which the membership is restricted to values of 0 (object is not a member of the set) and 1 (object is a member of the set).

Early versions of the cost minimization models (reference {5}) categorized each condition state into one of the following mutually exclusive categories:

Desirable
Undesirable
Neither Desirable nor Undesirable

Previously the B&SMS categorized as undesirable any condition state that had at least one element (e.g., for bridges - Deck, Superstructure, or Substructure) in Critical condition (Good, Fair, Poor, Critical are the possible levels). While one would surely agree that a bridge segment with deck, superstructure, and substructure all at the Critical level is an Undesirable condition state, it is not so clear cut with another segment with the Deck in Critical and the other two elements in Good. It is apparent that the former segment is more undesirable than the latter with only 1 element Critical. The previous performance constraints in reference {5} (also in the Arizona models [references {6,7}] upon which they are based) do not directly account for such distinctions.

Fuzziness is a natural result of the lack of well defined boundaries. An example would be the set of "rich" people. The transition between non-membership and membership for this set is gradual and lacks an obvious boundary. While there are clearly some individuals that are rich and would have a membership in this set equal to 1, there are many others for which it is not so clear. Lofti A. Zadeh in 1965 (reference {8}) published the initial work in this area. He set the groundwork for a fertile field that is seeing numerous applications including consumer products.

Confusion concerning fuzzy set theory often occurs because it is assumed to be related to probabilistic random variables or some form of uncertainty. Instead fuzziness is a result of the absence of sharply defined criteria of class membership. The fuzziness ensues from the vagueness or imprecision that results from the inability to adequately classify objects using conventional sets. Thus fuzzy sets are essential to properly address the true situation. Zadeh (reference {9}) has argued the following:

> *Indeed, fuzziness is more than a facet of reality; it is one of its most pervasive characteristics - a characteristic rooted in the bounded capacity of the human mind to process and store information.*

Categorizing a condition state into one of the three categories (Desirable, Undesirable, Neither Desirable nor Undesirable) was a difficult task. These are not "black and white" situations that are readily apparent. Each condition state within one of these three groupings was treated as having equal weight within that category, i.e., each condition state had a membership of 1 in the set it was placed and a membership of 0 in the other two sets.

In the optimization models presented here the condition states need not be treated as being a member of only one set as in the past. Rather each condition state has a membership in both the Desirable and Undesirable fuzzy sets. This membership $\Phi_{id}(s)$ (Desirable), $\Phi_{iu}(s)$ (Undesirable), may take any value on the range [0,1.0], i.e., $\Phi_{id}(s)$, $\Phi_{iu}(s) \in [0,1.0]$. An extremely desirable B&SMS condition state with all elements Good has $\Phi_{id}(s) = 1.0$ and $\Phi_{iu}(s) = 0$. Similarly, an extremely undesirable B&SMS condition state with all elements Critical has $\Phi_{id}(s) = 0.0$ and $\Phi_{iu}(s) = 1.0$. A similar situation holds for the PMS. Many condition states will have non-zero memberships in both the Desirable and Undesirable fuzzy sets. Additional details on the fuzzy set memberships may be found in reference {1}.

3. STEADY-STATE BENEFIT MAXIMIZATION

The steady-state models in the PMS and B&SMS are solved in order to set five year goals for the multi-year planning models. The model is given below. The summations over "i" cover the entire set of condition states for each stratum. Each "s" representing a stratum is unique. The **PMS/B&SMS steady-state model** uses the following variables:

Both Cost Minimization and Benefit Maximization

$w_{ia}(s)$	=	Proportion of units in stratum "s" that are in condition state "i" and receive action/scope "a". These are the decision variables.
$w^*_{ia}(s)$	=	Optimal output $w_{ia}(s)$.
$P_{iaj}(s)$	=	Probability of a segment transitioning in one year from condition state "i" to condition state "j" when action/scope "a" is applied in stratum "s".
$C_{ia}(s)$	=	Cost of action/scope "a" for a segment in stratum "s" in state "i".
$C^*(s)$	=	Optimal steady-state average segment cost for stratum "s". $C^*(s) = \sum_{i \in I(s)} \sum_{a \in M_i(s)} w^*_{ia}(s) C_{ia}(s)$
$N(s)$	=	Number of segments in stratum "s".
$I(s)$	=	Index set of conditions states "i" for stratum "s".
$M_i(s)$	=	Set of feasible actions/scopes for condition state "i" in stratum "s".
$p_k(s)$	=	Performance goal upper or lower bound for generalized performance constraint "k" of stratum "s".
$\Phi_{ik}(s)$	=	Generalized performance constraint parameter for condition state "i" - may be either fuzzy set memberships, $\Phi_{iu}(s)$ or $\Phi_{id}(s)$, or set to other values depending on the form of the generalized constraint "k" for stratum "s".
$\$_k(s)$	=	Stratum budget limits. These may be used to bound expenditures (upper or lower bound) in stratum "s" where $\$_k(s)$ is a specified budget limit.
S_P	=	Index set of PMS strata.
S_{BS}	=	Index set of B&SMS strata.
S_{P+BS}	=	Index set of PMS and B&SMS strata.

B_{BS} = Total annual budget for bridges and structures.

B_p = Total annual budget for pavement.

B_{P+BS} = Total annual budget for pavement, bridges and structures.

<u>Benefit Maximization objective function</u>

α = Lagrange multiplier used to move the budget constraint into objective function. This allows separation of the budget integrated optimization into individual stratum problems. The units of α for benefit maximization are (units of benefit) / (units of cost). It is unitless for multi-year cost minimization. <u>This is an output of the optimization process.</u> $\alpha \in [0.0, \infty)$.

$N_n(s)$ = Normalized number of segments in stratum "s". This is the proportion of segments in stratum "s" relative to the entire subsystem (either PMS or B&SMS).

$w_d(s)$ = The importance weight for being in desirable levels in stratum "s".

$w_u(s)$ = The importance weight for <u>not</u> being in undesirable levels in stratum "s".

$\Phi_{id}(s)$ = Desirable fuzzy set membership for condition state "i" in stratum "s".

$\Phi_{iu}(s)$ = Undesirable fuzzy set membership for condition state "i" in stratum "s".

$\pi_i(s)$ = Net worth of condition state "i" in stratum "s" that combines the individual desirable/not in undesirable importance weights, $w_d(s)$ and $w_u(s)$, with the desirable/undesirable fuzzy set membership functions, $\Phi_{id}(s)$ and $\Phi_{iu}(s)$ as follows:
$\pi_i(s) = w_d(s)\ \Phi_{id}(s) - w_u(s)\ \Phi_{iu}(s)$

ϕ_{sys} = Relative weight of subsystem.

ϕ_{sys} $= \phi_{B\&SMS}$ for a B&SMS stratum
$= \phi_{PMS}$ for a PMS stratum
$= \phi_{NPMS}$ for a NPMS stratum.

The **PMS and B&SMS steady-state** model is:

<u>Benefit Maximization Objective Function</u>

Maximize

$$N_n(s)\left[\phi_{sys} \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\ \pi_i(s)\right] - \alpha\ N(s) \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\ C_{ia}(s) \qquad (1)$$

<u>Cost Minimization Objective Function</u>

Minimize

$$N(s) \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s) C_{ia}(s) \qquad (2)$$

<u>Subject to (same constraints for benefit maximization or cost minimization):</u>

$$w_{ia}(s) \geq 0 \qquad \text{for all "i", "a", "s"} \qquad (3)$$

$$\sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s) = 1 \qquad \text{for all "s"} \tag{4}$$

$$\sum_{a \in M_j(s)} w_{ja}(s) - \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\, P_{iaj}(s) = 0 \quad \text{for all "j", "s"} \tag{5}$$

$$\sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\, \Phi_{ik}(s) \quad (\geq \text{ or } \leq) \quad p_k(s),\ k = 1, \ldots, K(s) \quad \text{for all "s"} \tag{6}$$

$$N(s) \sum_{i \in I(s)} \sum_{a \in M_i(s)} w^t_{ia}(s)\, C_{ia}(s) \ (\geq \text{ or } \leq) \ \$_k(s), \quad k = 1, \ldots, K_{BL}(s) \tag{7}$$

The benefit maximization objective function (1) maximizes a weighted sum reflecting benefits. The coefficient of $w_{ia}(s)$ is the product of several factors: normalized number of segments $N_n(s)$; $\Phi_{id}(s)$ and $\Phi_{iu}(s)$ that measure the degree of desirable/undesirable membership; importance weights $w_d(s)$ and $w_u(s)$; and the relative subsystem weight ϕ_{sys}. The weights $w_d(s)$ and $w_u(s)$ indicate the relative importance of the difference between proportions of strata in desirable conditions and the proportion not in undesirable conditions; the difference between functional classes; climatic differences; and bridge type (for bridge strata). For steady-state budget integration Equation (1) is summed over all strata as shown for B&SMS below to incorporate the budget constraint.

$$\sum_{s \in S_{BS}} N_n(s) [\phi_{sys} \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\, \pi_i(s)] - \alpha \sum_{s \in S_{BS}} N(s) \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s)\, C_{ia}(s) \tag{8}$$

The second (Lagrange) term of the benefit maximization objective function enforces the constraint below thus ensuring the budget (B_{P+BS}, B_P, or B_{BS}) is met. Lagrange relaxation is used since it permits the separation of the problem into an equivalent set of individual stratum models without having to actually specify the budget constraint. Each value of α corresponds to a given total budget level. This is a monotonic decreasing function that decrements at discrete levels of α.

$$\sum_{s \in S_{BS}} N(s) \sum_{i \in I(s)} \sum_{a \in M_i(s)} w_{ia}(s) C_{ia}(s) \leq (B_{BS},\ B_P,\ \text{or } B_{P+BS}) \tag{9}$$

(S_{BS} is replaced by S_P or S_{P+BS} as appropriate)

The cost minimization objective function (2) minimizes the cost in stratum "s". Constraints (3) and (4) ensure that solutions satisfy probability axioms. The $w_{ia}(s)$ are elements of a discrete joint probability distribution. Constraint (3) ensures the non-negativity (implicit in Linear Programming) of each individual element in this joint probability distribution while constraint (4) forces the sum over the feasible sample space (in a statistical sense) to equal 1. Constraints (5) are the steady-state equations for a Markov process (force the proportion of the network in condition state "i" to remain fixed, i.e., at steady state).

Constraints (6) are generalized performance constraints for each stratum (optional in benefit maximization, but necessary in cost minimization). These performance goal constraints allow considerable flexibility and bestow significant management control. Management may make detailed specific goals of relevance to them using these generalized performance constraints. Potential examples of the generalized performance constraints include constraints using fuzzy set goals or the older designations of desirable/undesirable goals. Another option is to set Element goals, e.g., % Decks wanted in at least Fair condition (or similar goals on distresses in PMS).

Equation (7) allows the optional inclusion of an upper or lower budget bound for an individual stratum. This is not normally used - usually the Lagrange term is used instead to control the entire network budget.

4. IMPORTANCE WEIGHTS

This section briefly covers the importance weights that are fully described in references {1,2,3}. These are multiplicative weights that are used to derive the w_d(s) and w_u(s) used in the PMS and B&SMS. They are developed within each subsystem (PMS, B&SMS) and then the weights across the subsystems are incorporated as well as weighing desirable versus undesirable.

Within the B&SMS the strata factors depend on whether the stratum is a bridge, culvert, or tunnel stratum. For bridges the stratum factors are bridge type, climate, and functional class. For culverts only climate and functional class are necessary. Tunnels have only 1 stratum and thus do not require any further breakdown.

Selected internal B&SMS ranking weights given below. The ranking weights (references {2,3}) have to be inverted to show importance.

<u>Selected B&SMS Ranking Weights</u>

Functional Class
Primary [2], Secondary [6]

Climate
Desert [1], Mountain [1], Coastal [.75]

Bridge type

Concrete slab - simple	[6]
Concrete slab - continuous	[6]
Concrete girders (or R.C. Box)	[6]
Steel composite	[8]
Prestressed girder	[4]
Prestressed box	[4]

Structure type
Bridges [3], Tunnels [3], Culverts [8]

Below is an example calculation of how the ranking weights above are converted to importance weights used in the optimization. This example deals only with the climatic aspect.

Desert = Coastal = 1/[1 + 1 + 1/.75] = .3
Mountain = (1/.75)/[1 + 1 + 1/.75] = .4

The PMS strata are based on Climate (3 levels) and functional class (3 levels). The same Climate weights employed for the B&SMS are also used for the PMS. For functional class, the ranking weights established were as follows:

PMS Functional Class
Primary [2], Secondary [4], and Feeder [8]

Below are selected intermediate importance weights that result from the material presented above. They are incorporated with additional weights (e.g., PMS vs. B&SMS, and Desirable vs. Undesirable) when the optimizations are run. The results are used in both the steady-state and multi-year optimizations.

In the list below the following are the six bridge types referred to:

1. Reinforced concrete slab bridges, simple span
2. Reinforced concrete bridges, continuous span

3. Prestressed girder (I, T, etc.) bridges (or reinforced concrete box girder bridges)
4. Steel composite bridges
5. Reinforced concrete T-girder bridges
6. Prestressed box girder bridges

BRIDGES

Func. Class	Climate	Bridge Type	Weight
Primary	Desert	1	1.31
Primary	Desert	4	.99
Primary	Desert	5	1.97
Primary	Mountain	1	1.75
Primary	Mountain	4	1.31
Primary	Mountain	5	2.63
Secondary	Desert	1	.44
Secondary	Desert	4	.33
Secondary	Desert	5	.66

PMS

Func. Class	Climate	Weight
Primary	Desert	1.54
Primary	Mountain	2.06
Secondary	Desert	.77
Secondary	Mountain	1.03
Feeder	Desert	.39
Feeder	Mountain	.51

5. MULTI-YEAR PMS/B&SMS OPTIMIZATION MODEL

Multi-year budget integration is a complex problem. The mathematical formulation of the multi-year model is not shown in this paper. The reader is referred to reference {10} for a complete listing and description of this model. The multi-year model develops a budget allocation such that the first-year budget is met while at the same time providing "smoothing" of the multi-year stratum budgets over the planning horizon leading to the desired steady-state goals. The first year budget can be achieved if sufficient relaxation of both the performance goals and budget targets is allowed.

6. GENERAL COMMENTS ON THE HMMS

The HMMS allows both cost minimization and benefit maximization. In cost minimization there is not a need for the many parameters introduced that in essence weight some aspect of pavement versus bridges. The coefficients used in the benefit maximization model presented here represent the specific values of one realization of this system - the Kingdom of Saudi Arabia. These values represent the combined interactive efforts of a multi-national task force overseen by the World Bank and its consultants. While such values are not always easy to obtain and agree upon, they do represent rationale trade-offs for estimating the significance of pavement versus bridges.

In the cost minimization mode, one can minimize cost with or without user cost (reference {11}). Thus the HMMS allows the minimization of agency cost or user cost in addition to the maximization of benefits as defined in this paper.

Each agency should evaluate its own set of parameters so that the weights are reflective of its values. The sensitivity of the results relative to the parameter values may be readily tested since the key parameters are used in the objective function. As an example, efficient parametric programming may easily determine the impact of changes in ϕ_{sys}.

The benefit maximization run shown in the next section was done for the Kingdom of Saudi Arabia. Many additional runs may be found in references {1, 2, 3}. The run shown in the next section had to meet specified performance goals.

Subject to meeting those goals it is clear that the benefit maximization wanted to allocate proportionally more additional funds to the bridge system when more money was available. In this example this is primarily due to bridges being weighted more heavily. It can be shown theoretically that the benefit maximization first year results asymptote as the Lagrange multiplier increases to a cost minimization (with the same performance goals). Thus the higher weighting of bridges versus pavement tends to shift supplemental funding (above the minimum needed to achieve the performance goals) to bridges in this particular case.

7. EXAMPLE RUN

Figure 1 graphs the total PMS and B&SMS network (all strata) budget as a function of the Lagrange multiplier α. The budget is a monotonically decreasing function of the Lagrange multiplier. As the budget is reduced the optimal mix across all bridge and pavement strata is determined. This ensures that the best use is made of the scarce resources available.

In this example the total budget decreases 71% over the range of the Lagrange multiplier shown. Most of this comes from a corresponding 76% reduction in the B&SMS budget while the PMS budget was reduced only 35%. These runs are based on multi-year benefit maximization. In all cases shown the performance goals specified for each stratum were met; however, since this was a benefit maximization run it attempted to achieve the most benefit possible. Benefit maximization when the Lagrange multiplier α equals zero corresponds to an unconstrained cost situation. Thus it is not surprising that the budget can be significantly reduced and still meet the performance goals. There is no significant drop in the total budget for values of the Lagrange multiplier larger than shown in Figure 1.

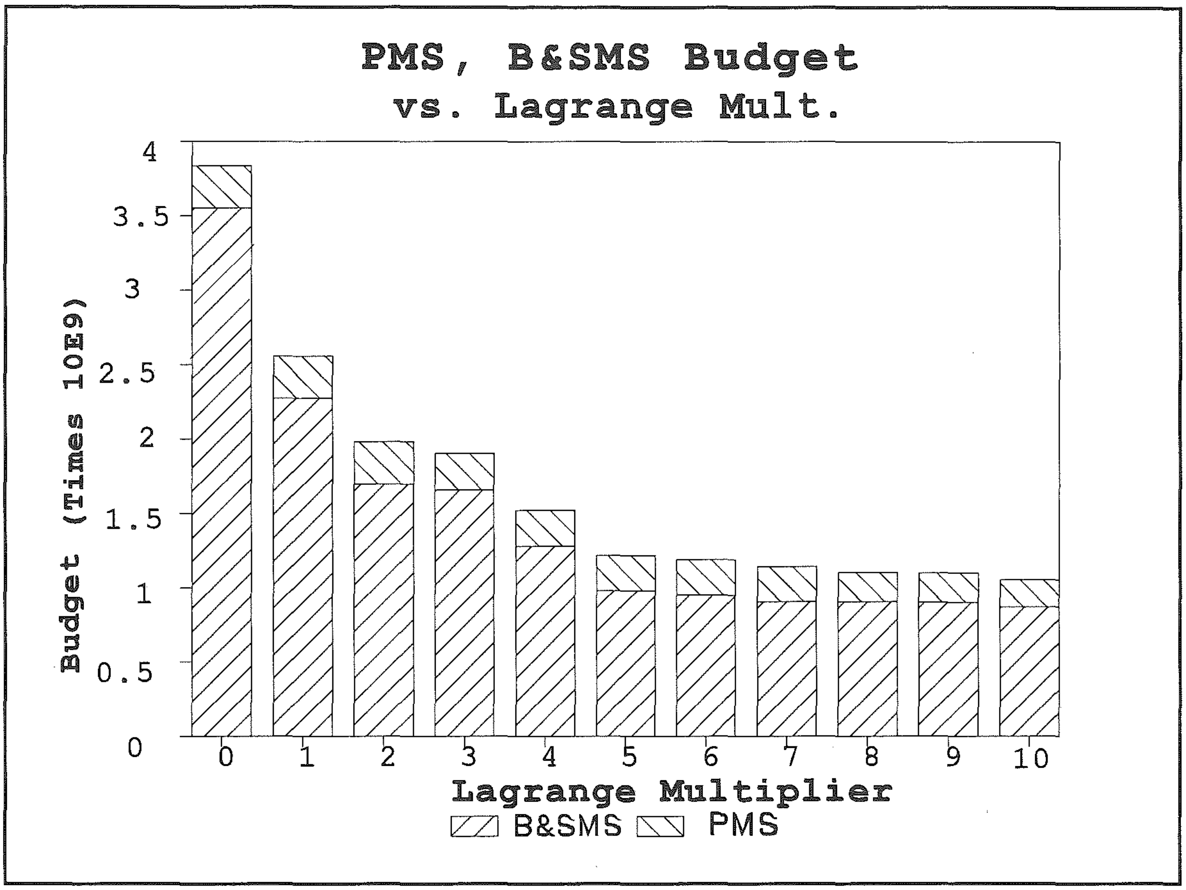

Figure 1 Total Budget as a function of the Lagrange Multiplier

8. SUMMARY

An integrated infrastructure management system has been presented. The major focus was on optimization models for steady-state and multi-year pavement and bridge management systems. These optimization models integrate the pavement and bridge management systems so that management can optimally allocate resources across the combined system. This integrated approach provides either cost minimization or benefit maximization options. In either case, optimal solutions are found for each stratum within the subsystems being linked. This system provides the potential for enhanced returns on scarce budgetary resources.

9. REFERENCES

1. Highways Maintenance Associates, "Highways Maintenance Management System - Stage F - Integration of Maintenance Budgets, Task F1/F2, Report on Framework of Integration Model, for the Ministry of Communications, Kingdom of Saudi Arabia, Riyadh, Saudi Arabia, November, 1993.

2. Highways Maintenance Associates, "Highways Maintenance Management System - Stage C - Pavement Management System, Task C2, Report on PMS Submodel Development," for the Ministry of Communications, Kingdom of Saudi Arabia, Riyadh, Saudi Arabia, September, 1993.

3. Highways Maintenance Associates, "Highways Maintenance Management System - Stage D - Bridges & Structures Management System, Task D3.2, Report on B&SMS Submodel Development," for the Ministry of Communications, Kingdom of Saudi Arabia, Riyadh, Saudi Arabia, November, 1993.

4. Harper, W.V., A. Al-Salloum, S. Al-Sayyari, S. Al-Theneyan, Jenny Lam, and Cheryl L. Helm. Selection of Ideal Maintenance Strategies in a Network Level Bridge Management System. In Transportation Research Record 1268, TRB, National Research Council, Washington, D.C., 1990, pp. 59-67.

5. Harper, W.V., J. Lam, A. al-Salloum, S. al-Sayyari, S. al-Theneyan, G. Ilves, and K. Majidzadeh. Stochastic Optimization Subsystem of a Network Level Bridge Management System. In Transportation Research Record 1268, TRB, National Research Council, Washington, D.C., 1990, pp. 68-74.

6. Kulkarni, R., Golabi, K., Finn, F., Alviti, E., and Nazareth, L., "Development of a Network Optimization System," Prepared for the Arizona Department of Transportation, Final Report, Volume I, May 1980.

7. Golabi, K., R.B. Kulkarni, and G.B. Way, "A Statewide Pavement Management System", Interfaces, Volume 12, December 6, 1982, pp. 5-21.

8. Zadeh, L. A., "Fuzzy Sets", Information and Control, volume 8, 1965, pp. 338-353.

9. Zadeh, L.A., "Fuzzy Set Theory - 'A Perspective'", Gupta, M.M., ed.; Saradis, G.N., and Gaines, B.R. , assoc. eds. Fuzzy Automata and Decision Processes, Elsevier North-Holland, New York, 1977, pp 3-4.

10. Harper, W.V., and Majidzadeh, K., "Integrated Pavement and Bridge Management Optimization", TRB Paper 93-0504, Transportation Research Board, Washington, D.C., 1993.

11. Harper, W.V., Majidzadeh, K., and B.B. Hurst, "Application of Expert Opinion in Customized Pavement Management Systems", Highways and Data Processing, pp. 549-557, Presses de l'ecole nationale des Ponts et chaussees, Paris, March 13-15, 1990.