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David G. Robertson  
*Otterbein University*

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## LECTURE 10

# PHYSICAL COUPLING SCHEMES AND QCD EXCLUSIVE PROCESSES

David G. Robertson

*Department of Physics  
The Ohio State University  
Columbus, OH 43210, USA*

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One of the most important problems in making reliable predictions in perturbative QCD is dealing with the dependence of the truncated perturbative series on the choice of renormalization scale  $\mu$  and scheme  $s$  for the QCD coupling  $\alpha_s(\mu)$ . Consider a physical quantity  $\mathcal{O}$ , computed in perturbation theory and truncated at next-to-leading order (NLO) in  $\alpha_s$ :

$$\mathcal{O} = \alpha_s(\mu) \left[ 1 + (A_1 + B_1 n_f) \frac{\alpha_s(\mu)}{\pi} + \dots \right], \quad (1)$$

where  $n_f$  is the effective number of quark flavors. The finite-order expression depends on both  $\mu$  and the choice of scheme used to define the coupling. In fact, Eq. (1) can be made to take on essentially any value by varying  $\mu$  and the renormalization scheme, which are *a priori* completely arbitrary. The scale/scheme problem is that of choosing  $\mu$  and the scheme  $s$  in an “optimal” way, so that an unambiguous theoretical prediction, ideally including some plausible estimate of theoretical uncertainties, can be made.<sup>(1)</sup>

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<sup>(1)</sup> The precise meaning of “optimal” in this context is connected to the minimization of remainders for the truncated series. As is well known, perturbation series in QCD are asymptotic, and thus there is an optimum number of terms that should be computed for a given observable. In general, very little is known about the remainders in pQCD; however, if we assume that pQCD series are sign-alternating, then the remainder can be estimated by the first neglected (or last included) term. This term

For any given observable there is no rigorously correct way to make this choice in general. However, a particular prescription may be supported to a greater or lesser degree by general theoretical arguments and, *a posteriori*, by its success in practical applications. From these perspectives, a particularly successful method for choosing the renormalization scale is that proposed by Brodsky, Lepage and MacKenzie [1]. In the BLM procedure, the renormalization scales are chosen such that all vacuum polarization effects from fermion loops are absorbed into the running couplings. A principal motivation for this choice is that it reduces to the correct prescription in the case of Abelian gauge theory. Furthermore, the BLM scales are physical in the sense that they typically reflect the mean virtuality of the gluon propagators. Another important advantage of the method is that it “pre-sums” the large and strongly divergent terms in the pQCD series which grow as  $n!(\alpha_s\beta_0)^n$ , i.e., the infrared renormalons associated with coupling constant renormalization.

Dependence on the renormalization scheme can be avoided by considering relations between physical observables only. By the general principles of renormalization theory, such a relation must be independent of any theoretical conventions, in particular the choice of scheme in the definition of  $\alpha_s$ . A relation between physical quantities in which the BLM method has been used to fix the renormalization scales is known as a “commensurate scale relation” (CSR) [2]. An important example is the generalized Crewther relation [2, 3], in which the radiative corrections to the Bjorken sum rule for deep inelastic lepton-proton scattering at a given momentum transfer  $Q$  are predicted from measurements of the  $e^+e^-$  annihilation cross section at a commensurate energy scale  $\sqrt{s} \propto Q$ .

In this talk I summarize recent applications of the BLM procedure to obtain CSRs relating QCD exclusive amplitudes to other observables, in particular the heavy quark potential  $V(Q^2)$  [4]. As we shall see, the heavy quark potential can be used to define a physical coupling scheme which is quite natural for perturbative calculations. It may also be useful in the context of a nonperturbative formulation of QCD based on light-cone quantization.

## BLM SCALE FIXING

The term involving  $n_f$  in Eq. (1) arises solely from quark loops in vacuum polarization diagrams. In QED these are the only contributions responsible for the running of the coupling, and thus it is natural to absorb them into the definition of the coupling. The BLM procedure is the analog of this approach in QCD. Specifically, we rewrite Eq. (1) in the form

$$\mathcal{O} = \alpha_s(\mu) \left[ 1 + \left( \frac{3\beta_0 B_1}{2} \right) \frac{\alpha_s(\mu)}{\pi} \right] \left[ 1 + \left( A_1 + \frac{33B_1}{2} \right) \frac{\alpha_s(\mu)}{\pi} \right], \quad (2)$$

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can take on essentially any value, however, by simply varying the scale and scheme, and thus its minimization is meaningless without invoking additional criteria.

correct to order  $\alpha_s^2$ , where  $\beta_0 = 11 - 2n_f/3$  is the lowest-order QCD beta function. The first term in square brackets can then be absorbed by a redefinition of the renormalization scale in the leading-order coupling, using

$$\alpha_s(\mu^*) = \alpha_s(\mu) \left[ 1 - \frac{\beta_0 \alpha_s(\mu)}{2\pi} \ln(\mu^*/\mu) + \dots \right]. \quad (3)$$

That is, the BLM procedure consists of defining the prediction for  $\mathcal{O}$  at this order to be

$$\mathcal{O} = \alpha_s(\mu^*) \left[ 1 + \left( A_1 + \frac{33B_1}{2} \right) \frac{\alpha_s(\mu^*)}{\pi} + \dots \right], \quad (4)$$

where

$$\mu^* \equiv \mu e^{3B_1}. \quad (5)$$

Note that knowledge of the NLO term in the expansion is necessary to fix the scale at LO. The scale occurring in the highest term in the expansion will in general be unknown. A natural prescription is to set this scale to be the same as that in the next-to-highest-order term.

A very important feature of this prescription is that  $\mu^*$  is actually independent of  $\mu$ . (This follows from considering the  $\mu$  dependence of  $B_1$ .) Thus pQCD predictions using the BLM procedure are unambiguous.

The same basic idea can be extended to higher orders, by systematically shifting  $n_f$  dependence into the renormalization scales order by order. The result is that a generic perturbative expansion

$$\frac{\alpha_s(\mu)}{\pi} + (A_1 + B_1 n_f) \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 + (A_2 + B_2 n_f + C_2 n_f^2) \left( \frac{\alpha_s(\mu)}{\pi} \right)^3 + \dots \quad (6)$$

is replaced by a series of the form

$$\frac{\alpha_s(\mu^*)}{\pi} + \tilde{A}_1 \left( \frac{\alpha_s(\mu^{**})}{\pi} \right)^2 + \tilde{A}_2 \left( \frac{\alpha_s(\mu^{***})}{\pi} \right)^3 + \dots \quad (7)$$

In general a different scale appears at each order in perturbation theory. In addition, the coefficients  $\tilde{A}_n$  are constructed to be independent of  $n_f$ , and so the form of the expansion is unchanged as momenta vary across quark mass thresholds. All effects due to quark loops in vacuum polarization diagrams are automatically incorporated into the effective couplings.

As discussed above, one motivation for this prescription is that it reduces to the correct result in the case of QED. In addition, when combined with the idea of commensurate scale relations (see below), the BLM method can be shown to be consistent with the generalized renormalization group invariance of Stückelberg and Peterman, in which one considers “flow equations” both in  $\mu$  and in the parameters that define the scheme.

To avoid scheme dependence, it is convenient to introduce physical effective charges, defined via some convenient observable, for use as an expansion parameter. An expansion of a physical quantity in terms of such a charge is a

relation between observables and therefore must be independent of theoretical conventions, such as the renormalization scheme, to any fixed order of perturbation theory. In practice, a CSR relating two observables  $A$  and  $B$  is obtained by applying BLM scale-fixing to their respective perturbative predictions in, say, the  $\overline{MS}$  scheme, and then algebraically eliminating  $\alpha_{\overline{MS}}$ .

A particularly useful scheme is furnished by the heavy quark potential  $V(Q^2)$ , which can be identified as the two-particle-irreducible amplitude for the scattering of an infinitely heavy quark and antiquark at momentum transfer  $t = -Q^2$ . The relation

$$V(Q^2) = -\frac{4\pi C_F \alpha_V(Q)}{Q^2}, \quad (8)$$

with  $C_F = (N_C^2 - 1)/2N_C = 4/3$ , then defines the effective charge  $\alpha_V(Q)$ . This coupling provides a physically-based alternative to the usual  $\overline{MS}$  scheme. Another useful charge is provided by the total  $e^+e^- \rightarrow X$  cross section, via the definition  $R(s) \equiv 3\Sigma e_q^2 (1 + \alpha_R(\sqrt{s})/\pi)$ . The CSR relating  $\alpha_V$  and  $\alpha_R$  is

$$\alpha_V(Q_V) = \alpha_R(Q_R) \left( 1 - \frac{25}{12} \frac{\alpha_R}{\pi} + \dots \right), \quad (9)$$

where the ratio of commensurate scales is  $Q_R/Q_V = e^{23/12 - 2\zeta_3} \simeq 0.614$  [4].

Physical couplings like  $\alpha_V$  are of course renormalization-group-invariant, i.e.  $\mu \partial \alpha_V / \partial \mu = 0$ . However, the dependence of  $\alpha_V(Q)$  on  $Q$  is controlled by an equation which is formally identical to the usual RG equation. Since  $\alpha_V$  is dimensionless we must have

$$\alpha_V = \alpha_V \left( \frac{Q}{\mu}, \alpha_s(\mu) \right). \quad (10)$$

Then  $\mu \partial \alpha_V / \partial \mu = 0$  implies

$$Q \frac{\partial}{\partial Q} \alpha_V(Q) = \beta_s(\alpha_s) \frac{\partial \alpha_V}{\partial \alpha_s} \equiv \beta_V(\alpha_V), \quad (11)$$

where

$$\beta_s = \mu \frac{\partial}{\partial \mu} \alpha_s(\mu). \quad (12)$$

This is formally a change of scheme, so that the first two coefficients  $\beta_0 = 11 - 2n_f/3$  and  $\beta_1 = 102 - 38n_f/3$  in the perturbative expansion of  $\beta_V$  are the standard ones.

## EXCLUSIVE AMPLITUDES AND $V(Q^2)$

Exclusive processes are particularly challenging to compute in QCD because of their sensitivity to the unknown nonperturbative bound state dynamics of the hadrons. However, there is an extraordinary simplification which occurs

when the hadrons are forced to absorb a large momentum transfer  $Q$ : one can separate the nonperturbative long-distance physics associated with hadron structure from the short-distance quark-gluon hard scattering amplitudes responsible for the dynamical reaction. A meson form factor, for example, factorizes to leading order in  $1/Q$  in the form

$$F_M(Q^2) = \int_0^1 dx \int_0^1 dy \phi_M(x, \tilde{Q}) T_H(x, y, Q^2) \phi_M(y, \tilde{Q}), \quad (13)$$

where  $\phi_M(x, \tilde{Q})$  is the meson distribution amplitude, which encodes the non-perturbative dynamics of the bound valence Fock state up to the resolution scale  $\tilde{Q}$ , and  $T_H$  is the leading-twist perturbatively-calculable amplitude for the subprocess  $\gamma^* q(x)\bar{q}(1-x) \rightarrow q(y)\bar{q}(1-y)$ , in which the incident and final mesons are replaced by valence quarks collinear up to the resolution scale  $\tilde{Q}$ . Contributions from nonvalence Fock states and the correction from neglecting the transverse momenta in the subprocess amplitude from the nonperturbative regime are higher twist, i.e., power-law suppressed. The transverse momenta in the perturbative domain lead to the evolution of the distribution amplitude in  $\tilde{Q}$  and to NLO corrections in  $\alpha_s$ . For further details and references see [5].

It is straightforward to obtain CSRs relating exclusive amplitudes to, e.g., the heavy quark potential. For the pion form factor, for example, we find [4]

$$F_\pi(Q^2) = \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) \frac{4\pi C_F \alpha_V(Q_V^*)}{(1-x)(1-y)Q^2} \left( 1 + C_V \frac{\alpha_V(Q_V^*)}{\pi} \right), \quad (14)$$

where  $C_V = -1.91$  and  $Q_V^{*2} = (1-x)(1-y)Q^2$  is the virtuality of the exchanged gluon in the underlying hard scattering amplitude. Eq. (14) represents a general connection between the form factor of a bound-state system and the irreducible kernel that describes the scattering of its constituents.

If we expand the QCD coupling about a fixed point [4], and assume that the pion distribution amplitude has the asymptotic form  $\phi_\pi(x) = \sqrt{3}f_\pi x(1-x)$ , with  $f_\pi \simeq 93$  MeV, then the integral over the effective charge in Eq. (14) can be performed explicitly. We thus find

$$Q^2 F_\pi(Q^2) = 16\pi f_\pi^2 \alpha_V(e^{-3/2}Q) \left( 1 - 1.91 \frac{\alpha_V}{\pi} \right) \quad (15)$$

for the asymptotic distribution amplitude. In addition

$$Q^2 F_{\gamma\pi}(Q^2) = 2f_\pi \left( 1 - \frac{5}{3} \frac{\alpha_V(e^{-3/2}Q)}{\pi} \right) \quad (16)$$

for the  $\gamma \rightarrow \pi^0$  transition form factor. A further prediction resulting from the factorized form of these results is that the normalization of the ratio

$$R_\pi(Q^2) \equiv \frac{F_\pi(Q^2)}{4\pi Q^2 |F_{\pi\gamma}(Q^2)|^2} \quad (17)$$

$$= \alpha_V(e^{-3/2}Q) \left( 1 + 1.43 \frac{\alpha_V}{\pi} \right) \quad (18)$$

is formally independent of the form of the pion distribution amplitude. The NLO correction given here assumes the asymptotic distribution amplitude.

A striking feature of these results is that the physical scale controlling the form factor in the  $\alpha_V$  scheme is very low:  $e^{-3/2}Q \simeq 0.22Q$ , reflecting the characteristic momentum transfer experienced by the spectator valence quark in lepton-meson elastic scattering. In order to compare these expressions to data, therefore, we require an Ansatz for  $\alpha_V$  at low scales. In Ref. [4] we consider a parameterization of the form

$$\alpha_V(Q) = \frac{4\pi}{\beta_0 \ln \left( \frac{Q^2 + 4m_g^2}{\Lambda_V^2} \right)}, \quad (19)$$

which effectively freezes  $\alpha_V$  to a constant value for  $Q^2 \leq 4m_g^2$ . A primary motivation for this is the observation that the data for exclusive amplitudes such as form factors, two-photon processes such as  $\gamma\gamma \rightarrow \pi^+\pi^-$ , and photoproduction at fixed  $\theta_{c.m.}$  are consistent with the nominal scaling of the leading-twist QCD predictions for momentum transfers down to a few GeV. This can be immediately understood if  $\alpha_V$  is slowly varying at low momentum. The scaling of the exclusive amplitude then follows that of the subprocess amplitude  $T_H$  with effectively fixed coupling.

The parameters  $\Lambda_V$  and  $m_g^2$  are determined by fitting to a lattice determination of  $V(Q^2)$  [6] and to a value of  $\alpha_R$  advocated in [7] using Eq. (9). We find  $\Lambda_V \simeq 0.16$  GeV and  $m_g^2 \simeq 0.2$  GeV<sup>2</sup>. With these values, the prediction for  $F_{\gamma\pi}$  is in excellent agreement with the data for  $Q^2$  in the range 2–10 GeV<sup>2</sup>. We also reproduce the scaling and normalization of the  $\gamma\gamma \rightarrow \pi^+\pi^-$  cross section at large momentum transfer. However, the normalization of the space-like pion form factor  $F_\pi(Q^2)$  obtained from electroproduction experiments is somewhat higher than that predicted by Eq. (15). This discrepancy may actually be due to systematic errors introduced by the extrapolation of the  $\gamma^*p \rightarrow \pi^+n$  data to the pion pole. What is at best measured in electroproduction is the transition amplitude between a mesonic state with an effective space-like mass  $m^2 = t < 0$  and the physical pion. It is theoretically possible that the off-shell form factor  $F_\pi(Q^2, t)$  is significantly larger than the physical form factor because of its bias towards more point-like  $q\bar{q}$  valence configurations in its Fock state structure. These and related issues are discussed elsewhere [8].

In any case, we find no compelling argument for significant higher-twist contributions in the few-GeV regime from the hard scattering amplitude or the endpoint regions, since such corrections would violate the observed scaling behavior of the data.

## SUMMARY

As we have emphasized, the  $\alpha_V$  scheme is quite natural when analyzing QCD processes perturbatively. By definition, it automatically incorporates quark

(as well as the corresponding gluon) vacuum polarization contributions into the coupling; thus the coefficients in a perturbative expansion do not change as momenta vary across quark mass thresholds. It is directly connected to one of the most useful observables in QCD, the heavy quark potential, which is accessible on the lattice as well as phenomenologically via the spectrum of heavy quarkonium. Finally, the scale-setting problem in QCD appears much less mysterious from this point of view: the scale appropriate for each appearance of  $\alpha_V$  in a Feynman diagram is just the momentum transfer of the corresponding exchanged gluon.<sup>(2)</sup> This prescription is equivalent to the BLM procedure.

It may also prove useful in the context of nonperturbative calculations based on the light-cone formalism. A light-cone Hamiltonian expressed in terms of  $\alpha_V$ , so that it reproduces covariant perturbation theory with  $\alpha_V$  appearing inside the momentum integrals, should be very well suited to studying, e.g., heavy quark systems: the effects of the light quarks and higher Fock state gluons that renormalize the coupling are already contained in  $\alpha_V$ . At high momentum scales  $\alpha_V$  should be computable via perturbation theory, while at low scales a semi-phenomenological Ansatz would be necessary. As we have discussed, exclusive processes can provide a valuable window for determining  $\alpha_V$  in the low-energy domain.

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<sup>(2)</sup> There are complications which arise when gluon self-couplings are present. These are discussed in [9].