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David G. Robertson
Otterbein University

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Composite operator renormalization and the trace anomaly

David G. Robertson

Department of Physics, University of California, Santa Barbara, CA 93106, USA

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The general connection between the renormalization of elementary fields and couplings and of appropriate composite operators is discussed. A general method is presented for computing the anomalous dimension matrix of the lagrangian composite operators, to all orders in perturbation theory, in terms of the elementary beta functions and anomalous dimensions. The trace anomaly for a general field theory is determined.

1. Introduction

Although the trace anomaly has been worked out in specific field theories, and a connection between the conventional renormalization group (RG) parameters and certain operator anomalous dimensions seems to be folklore, a precise and general formulation appears to be lacking. In this paper we relate the RG parameters of an arbitrary field theory to the anomalous dimension matrix of the lagrangian composite operators.

We begin by recalling standard techniques [1,2] for expressing the Green functions of the lagrangian composite operators in terms of Green functions of the elementary fields alone. This correspondence is displayed for a general field theory, and allows the computation of the divergences of the operator insertions in terms of the conventional RG parameters. The result is a simple recipe for calculating, to all orders in perturbation theory, the anomalous dimensions of the lagrangian operators in terms of the ordinary beta functions and anomalous dimensions.

These anomalous dimensions are of interest for several reasons. The lagrangian operators can of course appear in various operator product expansions; their anomalous dimensions control the effective scale dependence of the associated coefficient functions. In addition, the anomalous trace of the energy-momentum tensor (EMT) is necessarily built up out of those operators appearing in the lagrangian. Understanding the general relation between the op-

erator renormalization matrix and the conventional RG parameters allows us to determine the trace anomaly for an arbitrary field theory. This computation is presented in section 3, along with some remarks concerning multiplicatively renormalized and renormalization group-invariant operators, and the Callan-Symanzik equation.

2. Renormalization of the lagrangian composite operators

Throughout this paper shall we employ dimensional regularization, following the conventions of ref. [3].

We consider a generic theory of a set of fields $\{\phi_\alpha\}$ in d_0 (continued to $d_0 - 2\epsilon$) spacetime dimensions. These may be scalar, spinor or gauge fields, under certain caveats to be detailed below. The lagrangian consists of suitable kinetic operators K_α and a number of interaction terms V_i , with couplings denoted g_i . The vertex term V_i contains p_α^i powers of the field ϕ_α or its derivative, and the couplings are assumed for simplicity to be dimensionless; mass terms or other dimensionful couplings are easily incorporated into the analysis.

Now, following e.g. the presentation of ref. [2] we can obtain functional differential equations relating the generating functionals of 1PI Green functions with a single zero-momentum insertion of K_α or V_i to the generating functional of elementary 1PI Green func-

tions $\Gamma[\phi;g]$. For insertions of the kinetic operators K_α we have

$$i\Gamma_{K_\alpha}[\phi;g] = \left(\frac{1}{2} \int d^6x \phi_\alpha \frac{\delta}{\delta\phi_\alpha} - \frac{1}{2} \sum_i p_\alpha^i g_i \frac{\partial}{\partial g_i} \right) \Gamma[\phi;g], \quad (2.1)$$

while for the vertex operators V_i we find

$$i\Gamma_{V_i}[\phi;g] = g_i \frac{\partial}{\partial g_i} \Gamma[\phi;g]. \quad (2.2)$$

Although these equations may be straightforwardly obtained using the (nonperturbative) techniques of ref. [2], we can understand them easily in terms of Feynman graphs. In eq. (2.1), the insertion of K_α essentially counts the number of ϕ_α lines in the graph at a given order of perturbation theory. This is achieved by the second term on the RHS of (2.1) (the factor $\frac{1}{2}$ is to avoid double counting, since each internal line connects two vertices); the first term removes the insertions from external lines. For the vertex operators the insertion simply counts the number of elementary vertices of the appropriate type.

Let us demonstrate how eqs. (2.1) and (2.2) may be used to calculate the operator anomalous dimensions in a specific example. We consider the massless scalar ϕ^3 theory in six (continued to $6-2\epsilon$) space-time dimensions, specified by the lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{g\phi^3}{3!}. \quad (2.3)$$

This theory is fairly sick, but is a useful toy. We denote the lagrangian operators by $K \equiv \frac{1}{2} (\partial_\mu \phi)^2$ and $V \equiv (g/3!) \phi^3$, and make the general remark that for a renormalizable theory the set of lagrangian operators at zero momentum is closed under renormalization^{#1}.

We begin by writing the effective action as an expansion in powers of $(\partial_\mu \phi)$:

$$\Gamma^0[\phi_0;g] = \int d^6x \left(A \cdot \frac{1}{2} (\partial_\mu \phi_0)^2 - B \frac{g\phi_0^3}{3!} + \dots \right). \quad (2.4)$$

^{#1} This is not generally the case for gauge theories, but is true when quantized in background field gauge; these issues are discussed further below.

Here ϕ_0 and g^0 are the bare field and coupling, respectively, and A and B are some (divergent) functions of g^0 . The renormalized effective action takes the form

$$\Gamma^r[\phi;g] \equiv \Gamma^0[Z^{1/2}\phi;Z_g g] = \int d^6x \left(AZ \cdot \frac{1}{2} (\partial_\mu \phi)^2 - BZ_g Z^{3/2} \frac{g\phi^3}{3!} + \dots \right), \quad (2.5)$$

where we have introduced the renormalized field ϕ and coupling g defined by $\phi_0 = Z^{1/2}\phi$ and $g^0 = Z_g g$. Renormalizing the theory consists of choosing Z and Z_g so that $\Gamma^r[\phi;g]$ is rendered finite; we shall define our renormalization scheme so that $AZ = BZ_g Z^{3/2} = 1$. In addition, operator renormalization constants Z_{ab} are defined by the requirement that the renormalized generating functionals for operator insertions, defined by

$$\Gamma_{\mathcal{O}_a}^r = Z_{ab}^{-1} \Gamma_{\mathcal{O}_b}^0[Z^{1/2}\phi;Z_g g], \quad (2.6)$$

be finite (here \mathcal{O}_a means $\{K, V\}$ for $a = \{1, 2\}$).

Now, we can use eqs. (2.1) and (2.2) to compute the operator effective actions on the RHS of (2.6). For example, eq. (2.1) gives

$$i\Gamma_K^0 = \left(\frac{1}{2} \int d^6y \phi_0(y) \frac{\delta}{\delta\phi_0(y)} - \frac{3}{2} g^0 \frac{\partial}{\partial g^0} \right) \times \int d^6x \left(A \cdot \frac{1}{2} (\partial_\mu \phi_0)^2 - B \frac{g^0 \phi_0^3}{3!} + \dots \right) = \int d^6x \left((1 - \frac{3}{2}A') \cdot \frac{1}{2} (\partial_\mu \phi)^2 + \frac{3}{2}B' \frac{g\phi^3}{3!} + \dots \right), \quad (2.7)$$

where

$$A' \equiv g^0 \frac{\partial}{\partial g^0} \ln A \quad (2.8)$$

and similarly for B . Eq. (2.2) likewise gives

$$i\Gamma_V^0 = \int d^6x \left(-A' \cdot \frac{1}{2} (\partial_\mu \phi)^2 + (1+B') \frac{g\phi^3}{3!} + \dots \right). \quad (2.9)$$

Eqs. (2.7) and (2.9) then allow us to express the requirement that the RHS of (2.6) be finite as a matrix equation:

$$Z^{-1}W = \text{finite}, \quad (2.10)$$

where

$$W \equiv \begin{pmatrix} 1 - \frac{3}{2}A' & \frac{3}{2}B' \\ -A' & 1 + B' \end{pmatrix}. \quad (2.11)$$

A particular choice for the matrix of constants appearing on the RHS of (2.10) is a choice of renormalization prescription for the operator counterterms. We choose the 2×2 unit matrix, so that, trivially, $Z = W$. We note that the elements of W can generally be read off quite simply from the lagrangian. We shall return to exploit this below.

In dimensional regularization the operator anomalous dimensions are determined by the simple poles in the Z_{ab} (see ref. [3]):

$$\Gamma_{ab} = -g \frac{\partial}{\partial g} Z_{ab}^{(1)}, \quad (2.12)$$

where

$$Z_{ab} \equiv \delta_{ab} + \sum_i \frac{Z_{ab}^{(i)}}{\epsilon^i}.$$

Now, in obtaining the pole part of eq. (2.11) we note that in minimal subtraction the difference between g^0 and g is itself of order $1/\epsilon$. We may therefore neglect this distinction to obtain, for example

$$\text{Res} \left\{ g^0 \frac{\partial}{\partial g^0} \ln Z \right\} = g \frac{\partial Z^{(1)}}{\partial g}, \quad (2.13)$$

where $Z^{(1)}$ is the residue of the simple pole in Z . With the standard expressions [3] for the ordinary beta function and anomalous dimension:

$$\beta = g^2 \frac{\partial}{\partial g} Z_g^{(1)}, \quad \gamma = -\frac{1}{2} g \frac{\partial}{\partial g} Z^{(1)}, \quad (2.14)$$

we thus arrive at the anomalous dimension matrix:

$$\Gamma_{ab} = \begin{pmatrix} 3g \frac{\partial \gamma}{\partial g} & \frac{3}{2}g \frac{\partial}{\partial g} \left(\frac{\beta}{g} \right) - \frac{9}{2}g \frac{\partial \gamma}{\partial g} \\ 2g \frac{\partial \gamma}{\partial g} & g \frac{\partial}{\partial g} \left(\frac{\beta}{g} \right) - 3g \frac{\partial \gamma}{\partial g} \end{pmatrix}. \quad (2.15)$$

The application of this type of analysis to non-abelian gauge theories deserves some additional comment, as there are certain subtleties associated with the renormalization of composite operators in these theories. Briefly stated, the renormalization of gauge-invariant operators requires the introduction of gauge-noninvariant counterterms. Furthermore, the

computation of the anomalous dimensions of gauge-invariant operators requires that this noninvariant mixing be included in general [1,4]. This will typically make the computation of the renormalization matrix prohibitively difficult (because the number of gauge-noninvariant operators whose mixing must be considered is usually huge) unless certain tricks are employed. The simplest of these tricks is to quantize the theory in the background field (BF) gauge [5]. In this formalism the gauge invariance of the theory is essentially manifest and no such noninvariant mixing occurs.

This was the procedure used in ref. [2] to compute the anomalous dimension of the operator $F^2 \equiv F_{\mu\nu}^a F^{a\mu\nu}$. The residual gauge invariance allows the effective action to be expressed as a gauge-invariant functional of the gauge fields only; this is a much stronger constraint on the structure of the counterterms than the usual BRST invariance alone. We emphasize here that this issue is purely one of calculational convenience. The analysis of ref. [2] could be pushed through in a covariant gauge, but various other operators would have to be included (including e.g., ghost operators [1,4]). By working in BF gauge, we avoid these complications; operator renormalization in gauge theory is then not particularly different from any other field theory, and the above techniques may be applied directly.

As a simple application of the above results, let us now compute the trace and determinant of the anomalous dimension matrix for our arbitrary theory. The trace turns out to have a pleasant expression in terms of the beta functions alone.

From eq. (2.1) we find that each kinetic operator K_α contributes to Z a diagonal term $Z_{\alpha\alpha}$ given by

$$Z_{\alpha\alpha} = 1 + \frac{1}{2} \sum_i p_\alpha^i g_i^0 \frac{\partial}{\partial g_i^0} \ln Z_\alpha, \quad (2.16)$$

where Z_α is the wavefunction renormalization constant for ϕ_α . (We assume the previous renormalization convention, in which the functions appearing in the effective action times the relevant field and coupling renormalization factors are all equal to unity; see, e.g., eq. (2.5).) Each vertex operator V_i likewise provides a term

$$Z_{ii} = 1 - \left(g_i^0 \frac{\partial}{\partial g_i^0} \ln Z_{g_i} + \frac{1}{2} \sum_\alpha p_\alpha^i g_i^0 \frac{\partial}{\partial g_i^0} \ln Z_\alpha \right), \quad (2.17)$$

where Z_{g_i} is the coupling renormalization constant for g_i . Eqs. (2.16) and (2.17) combine to give

$$\text{Tr}(\mathbf{Z}-\mathbf{1}) = - \sum_i g_i^0 \frac{\partial}{\partial g_i^0} \ln Z_{g_i}. \quad (2.18)$$

As before, in extracting the pole part of eq. (2.18) we can neglect the distinction between bare and renormalized couplings to obtain

$$\text{Res}\{\text{Tr}(\mathbf{Z}-\mathbf{1})\} = - \sum_i g_i \frac{\partial Z_{g_i}^{(1)}}{\partial g_i}. \quad (2.19)$$

Now, if we denote by $(d_i\epsilon)$ the mass dimension of g_i^0 in $d_0-2\epsilon$ dimensions, that is, $g_i^0 = \mu^{d_i\epsilon} Z_{g_i} g_i$, then the RG parameters are given by

$$\beta_i = g_i \sum_k d_k g_k \frac{\partial Z_{g_i}^{(1)}}{\partial g_k}, \quad (2.20)$$

$$\gamma_\alpha = -\frac{1}{2} \sum_k d_k g_k \frac{\partial Z_\alpha^{(1)}}{\partial g_k}. \quad (2.21)$$

Similarly, for the operator anomalous dimensions we have

$$\Gamma_{ab} \equiv Z_{ac}^{-1} \left(\mu \frac{\partial}{\partial \mu} Z_{cb} \right) = - \sum_k d_k g_k \frac{\partial Z_{ab}^{(1)}}{\partial g_k}. \quad (2.22)$$

Eqs. (2.19), (2.20) and (2.22) now combine to yield

$$\text{Tr} \Gamma = \sum_i g_i \frac{\partial}{\partial g_i} \left(\frac{\beta_i}{g_i} \right). \quad (2.23)$$

This result holds for arbitrary theories of scalars, fermions, and gauge fields, with the understanding that BF gauge is to be employed.

The determinant of Γ is also readily computed. As we shall see below, there always exist linear combinations of the lagrangian operators, closely related to the equations of motion, which are finite and so have zero anomalous dimension. The existence of zero eigenvalues of Γ implies $\det \Gamma = 0$.

3. Multiplicatively renormalized operators and the trace anomaly

We can also search for linear combinations of the lagrangian operators which are multiplicatively renormalized or RG-invariant. For instance, it is easy to check that the operator $(2K-3V)$ in the ϕ_3^3 theory has zero anomalous dimension. At zero momentum

this object is related to the equation of motion for ϕ ; more precisely

$$2K-3V = \int d^6x \phi(x) \frac{\delta \mathcal{S}}{\delta \phi(x)} \quad (3.10)$$

(after an integration by parts), where \mathcal{S} is the action. The finiteness of this operator follows from the fact that it is essentially the canonical trace of the EMT.

To construct the corresponding operators for the general theory, we expand eqs. (2.1) and (2.2) in powers of the fields ϕ_α . This gives relations between the 1PI n_α -point functions:

$$2i\Gamma_{K_\alpha}^{(n_\beta)0} = \left(n_\alpha - \sum_i p_\alpha^i g_i^0 \frac{\partial}{\partial g_i^0} \right) \Gamma^{(n_\beta)0}, \quad (3.2)$$

$$i\Gamma_{V_i}^{(n_\beta)0} = g_i^0 \frac{\partial}{\partial g_i^0} \Gamma^{(n_\beta)0}, \quad (3.3)$$

where $\Gamma^{(n_\beta)}$ stands for $\Gamma^{(n_1, n_2, \dots)}$. Eqs. (3.2) and (3.3) combine to yield e.g.

$$2\Gamma_{K_\alpha}^{(n_\beta)0} + \sum_i p_\alpha^i \Gamma_{V_i}^{(n_\beta)0} = i n_\alpha \Gamma^{(n_\beta)0}. \quad (3.4)$$

Now, since the RHS of (3.4) is made finite by elementary wavefunction and coupling renormalizations alone, the LHS must be made finite by these as well. Thus these combinations of operators are not renormalized. At zero momentum these operators are essentially the equations of motion for the fields ϕ_α .

We expect that there will be one other RG-invariant operator, namely the anomalous trace of the EMT. That this object should be invariant follows from the general properties of conserved currents: these must satisfy the nonlinear commutation relations of the associated symmetry group so that their normalization is fixed. For the ϕ_3^3 theory it is simple to verify that the combination

$$\Theta = -2\gamma[K] + \left(3\gamma - \frac{\beta}{g} \right) [V] \quad (3.5)$$

satisfies $\mu \partial \Theta / \partial \mu = 0$ ^{#2}. Here the square brackets denote renormalized composite operators, defined by $[\mathcal{O}_a] = Z_{ab}^{-1} \mathcal{O}_b^0$ (see eq. (2.6)). This is indeed the correct trace anomaly for this theory, as may be ver-

^{#2} It is also easy to check that this is the *only* other RG-invariant linear combination of the lagrangian operators in the theory.

ified by explicitly constructing the EMT and taking its trace.

In fact, we can perform this computation for the general theory almost as easily. We begin with the standard Noether expression for the EMT [6]:

$$\Theta_{\mu\nu} = \sum_{\alpha} \left[\frac{\partial \mathcal{L}}{\partial(\partial^{\mu}\phi_{\alpha})} (\partial_{\nu}\phi_{\alpha}) \right] - g_{\mu\nu} \mathcal{L}^0. \quad (3.6)$$

Note that although the zero-momentum insertions of $\Theta_{\mu\nu}$ are finite (after wavefunction and coupling constant renormalization), its constituents can be divergent. Taking the trace of (3.6), with $g_{\mu\nu}g^{\mu\nu} = d_0 - 2\epsilon$, we obtain

$$\Theta = \sum_{\alpha} \left[\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi_{\alpha})} (\partial_{\mu}\phi_{\alpha}) \right] - (d_0 - 2\epsilon) \mathcal{L}^0.$$

The term proportional to ϵ is the anomaly:

$$\Theta^{\text{an.}} = 2\epsilon \mathcal{L}^0. \quad (3.7)$$

This is to be regarded as a relation between operator insertions. Now $\mathcal{L} \equiv \sum_a \mathcal{O}_a$, where a indexes both the $\{K_{\alpha}\}$ and the $\{V_i\}$. Renormalization results in

$$\Theta^{\text{an.}} = 2 \sum_a [\mathcal{O}_a] \left(\sum_b Z_{ab}^{(1)} \right). \quad (3.8)$$

The calculation of the pole part of $\sum_b Z_{ab}$ is readily accomplished using eqs. (2.1) and (2.2). We shall denote the divergent function appearing with the operator \mathcal{O}_a in the expansion of the effective action by A_a (recall eq. (2.4)) and employ the usual renormalization conditions, that A_a times the relevant field and coupling renormalization factors are all equal to unity.

Now, if b indexes one of the kinetic operators K_{α} , then from eq. (2.1) we get a contribution

$$Z_{\alpha a} \approx -\frac{1}{2} \sum_i p'_{\alpha} g_i^0 \frac{\partial}{\partial g_i^0} \ln A_a, \quad (3.9)$$

where we have ignored an irrelevant (finite) term. If b indexes one of the V_i then we obtain

$$Z_{i a} = g_i^0 \frac{\partial}{\partial g_i^0} \ln A_a. \quad (3.10)$$

Combining eqs. (3.9) and (3.10) results in

$$\sum_b Z_{ba} = -\frac{1}{2} \sum_i (t_i - 2) g_i^0 \frac{\partial}{\partial g_i^0} \ln A_a,$$

where $t_i \equiv \sum_{\alpha} p'_{\alpha}$ is the total number of lines flowing

into the vertex V_i . Now, in $d_0 - 2\epsilon$ dimensions the mass dimension d_i of the coupling g_i^0 is just $(t_i - 2)$. Thus

$$\sum_b Z_{ba} = -\frac{1}{2} \sum_i d_i g_i^0 \frac{\partial}{\partial g_i^0} \ln A_a. \quad (3.11)$$

Consider the case when \mathcal{O}_a is one of the K_{α} . Then $A_a = Z_{\alpha}^{-1}$ and

$$\sum_b Z_{b\alpha}^{(1)} = \frac{1}{2} \sum_i d_i g_i \frac{\partial}{\partial g_i} Z_{\alpha}^{(1)} = -\gamma_{\alpha} \quad (3.12)$$

by eq. (2.21). Similarly, when \mathcal{O}_a is one of the V_i then $A_a = Z_{g_i}^{-1} (\prod_{\alpha} Z_{\alpha}^{-p'_{\alpha}/2})$ and we obtain

$$\sum_b Z_{bi}^{(1)} = -\frac{1}{2} \sum_{\alpha} p'_{\alpha} \gamma_{\alpha} + \frac{\beta_i}{2g_i}. \quad (3.13)$$

Inserting eqs. (3.12) and (3.13) into (3.8) we have

$$\Theta^{\text{an.}} = -2 \sum_{\alpha} \gamma_{\alpha} [K_{\alpha}] + \sum_i \left(-\sum_{\alpha} p'_{\alpha} \gamma_{\alpha} + \frac{\beta_i}{g_i} \right) [V_i]. \quad (3.14)$$

For the ϕ_s^3 theory this is precisely eq. (3.5).

As another example, consider pure $SU(N)$ gauge theory. The lagrangian has the schematic form

$$-\frac{1}{4} (F_{\mu\nu}^a)^2 \sim (\partial A)^2 + g(\partial A)A^2 + g^2 A^4.$$

We can disregard the gauge-fixing and ghost operators by considering only on-shell amplitudes. Now, in BF gauge β and γ are related through $\gamma = \beta/g$, so that each term in the lagrangian gets a factor $-2\beta/g$. Thus

$$\Theta^{\text{an.}} = + \frac{\beta}{2g} [F^2],$$

which is the correct result for on-shell insertions [7]. (Actually there is a slight difference between our Noether EMT and the EMT which occurs in ref. [7]. However, the extra terms may be shown to make no contribution to the anomaly [8].)

Note that in general certain dimension d_0 operators *do* survive at a RG fixed point but that by eq. (3.4) these operators are finite.

It should perhaps be emphasized that eq. (3.14) holds for zero-momentum insertions of $\Theta^{\text{an.}}$ only. In some cases, however, it may be extended to arbitrary momentum transfer. This is possible if the same set of lagrangian operators considered at $q=0$ also forms a closed set of operators under renormalization for

$q \neq 0$. This has been established for gauge theories [7], but is not true in general. For example, in the ϕ^3 theory the operator $(\phi \partial^2 \phi)$ mixes with K and V at $q \neq 0$. (This operator differs from K only by a total divergence and so may be disregarded for $q=0$.) Hence it may appear in the trace anomaly at nonzero momentum.

We conclude with a remark concerning the relevance of the trace anomaly to the study of the scale dependence of renormalized field theories. Making use of the renormalized versions of eqs. (3.2) and (3.3) the Callan–Symanzik equation for the general theory may be written in the form

$$\sigma \frac{\partial}{\partial \sigma} \Gamma^{(n_\alpha)}(\sigma p, g_i) = i \Gamma_{\Theta^{an}}^{(n_\alpha)}(0; \sigma p, g_i) + \left(d_0 - \sum_{\alpha} d_{\alpha} n_{\alpha} \right) \Gamma^{(n_\alpha)}(\sigma p, g_i). \quad (3.15)$$

Here d_{α} is the canonical mass dimension of the field ϕ_{α} . In this context the trace anomaly serves to generate the anomalous (i.e., noncanonical) terms in the scaling derivative of Green functions.

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References

- [1] H. Kluberg-Stern and J.B. Zuber, *Phys. Rev. D* 12 (1975) 467, 3159.
- [2] B. Grinstein and L. Randall, *Phys. Lett. B* 217 (1989) 335.
- [3] D.J. Gross, Applications of the renormalization group to high-energy physics, in: *Methods in field theory*, eds. R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976).
- [4] S. Joglekar and B.W. Lee, *Ann. Phys.* 97 (1976) 160.
- [5] B.S. DeWitt, *Phys. Rev.* 162 (1967) 1195, 1239; G. 't Hooft, *Nucl. Phys. B* 62 (1973) 444; and in: *Functional and probabilistic methods in quantum field theory*, XII Winter School of Theoretical Physics (Karpacz, 1975), Vol. 1, Acta Universitatis Wratislavenensis no. 38; an especially useful reference is L.F. Abbot, *Nucl. Phys. B* 185 (1981) 189.
- [6] C. Itzykson and J.B. Zuber, *Quantum field theory* (McGraw-Hill, New York, 1980).
- [7] J.C. Collins, A. Duncan and S.D. Joglekar, *Phys. Rev. D* 16 (1977) 438; N.K. Nielsen, *Nucl. Phys. B* 120 (1977) 212.
- [8] D.G. Robertson, Anisotropic scaling in relativistic field theory, UCSB preprint UCSBTH-89-58, *Intern. J. Mod. Phys. A*, to appear.