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θ vacua in the light-cone Schwinger model

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Abstract

We discuss the bosonized Schwinger model in light-cone quantization, using discretization as an infrared regulator. We consider both the light-cone Coulomb gauge, in which all gauge freedom can be removed and a physical Hilbert space employed, and the light-cone Weyl (temporal) gauge, in which the Hilbert space is unphysical and a Gauss law operator is used to select a physical subspace. We describe the different ways in which the θ vacuum is manifested depending on this choice of gauge, and compute the θ -dependence of the chiral condensate in each case.

1. Introduction

The method of Discretized Light-Cone Quantization (DLCQ) [1] has recently become a viable non-perturbative tool for studying quantum field theories, especially in two space-time dimensions, but possibly also in four [2]. It neatly unites the advantages of an infrared regulated framework and the vacuum simplicity of Dirac's "front form" of relativistic dynamics [3], and has been applied to a variety of toy models with considerable success.

The simplicity of the vacuum is a major advantage of the light-cone approach [4]. It is also a puzzle, however, particularly in light of the nontrivial physics associated with, e.g., the QCD vacuum. It is therefore important to understand how physics that is normally related to the vacuum appears in the light-cone framework. In DLCQ, any vacuum structure must necessarily be connected with the $k^+ = 0$ Fourier modes

of the fields². There has recently been a great deal of effort devoted to studying the properties of these zero modes, which can be quite nontrivial [5]. This work has shown that some types of vacuum structure – spontaneous breaking of discrete symmetries in scalar field theories, for example – is in fact recovered with a careful treatment of the zero modes.

The purpose of this note is to discuss the connection between zero modes and another type of vacuum structure: the θ vacuum. We shall address this in the simplest nontrivial setting, namely the Schwinger model [6]. This model has been discussed extensively in the light-cone literature, mainly in the fermionic representation. McCartor in particular has given a thorough treatment of the fermionic version [7]. There are many subtleties that must be addressed in order to understand the vacuum structure from this point of view – the left-moving fermions, the proper definition of operator products, and the selection of a suitable physical subspace, to name a few. The anomaly rela-

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² This follows from simple kinematical considerations.

tion is also bound up in the subtleties, and is closely connected to the vacuum structure.

Here we shall sidestep most of these difficulties by studying the bosonized form of the theory. In this case the anomaly relation, for example, is obtained directly as an equation of motion. In addition, the condition that physical states be chargeless is automatically implemented through the bosonization. The only non-trivial issue that remains is the θ structure, which can thus be studied in isolation. We shall focus in particular on the θ -dependence of the chiral condensate $\langle \theta | \bar{\psi} \psi | \theta \rangle$ in this model. This represents the only “observable” consequence of the θ vacuum [6]. We shall study this dependence in two different gauges – the light-cone Weyl gauge, in which we have an extended Hilbert space, and the light-cone Coulomb gauge, in which we eliminate all gauge freedom at the classical level. That the precise manifestation of the topological structure can be gauge-dependent is well known [8]. Our aim is to exhibit the gauge-dependence of the θ -structure in light-cone quantization. As we shall see, the correct results are obtained, but in a somewhat different way than in the conventional approach.

2. Canonical formalism

Our starting point is the bosonized form of the Schwinger model Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - gA_\mu \epsilon^{\mu\nu} \partial_\nu \phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\epsilon^{\mu\nu}$ is completely antisymmetric with $\epsilon^{01} = +1$. Some useful correspondences with the fermionic version of the model are $g = e/\sqrt{\pi}$, relating the mass of the Schwinger boson to the gauge coupling e , and

$$\bar{\psi} \psi = K : \cos(2\sqrt{\pi} \phi) : , \quad (2)$$

where $K = ge^{\gamma_E}/2\pi$ with γ_E the Euler–Mascheroni constant.

We choose the light-cone convention $x^\pm = (x^0 \pm x^1)/\sqrt{2}$, and quantize independent fields on the line $x^+ = 0$. We take space to be a finite interval, $-L \leq x^- \leq L$, with periodic boundary conditions on the fields. It is therefore important to distinguish the zero and normal mode parts of fields in a Fourier expansion.

We adopt the notation $\phi(x^-) = \phi_0 + \varphi(x^-)$ and $A^\mu(x^-) = A_0^\mu + A_n^\mu(x^-)$, with

$$\phi_0 = \frac{1}{2L} \int_{-L}^L dx^- \phi \quad (3)$$

and analogously for A_0^μ . When a given expression holds for both zero and normal modes, however, the distinction will be suppressed. In addition, the periodic delta function with its zero mode subtracted will be needed. We define

$$\bar{\delta}(x) \equiv \delta(x) - \frac{1}{2L}, \quad (4)$$

where

$$\delta(x) = \frac{1}{2L} \sum_{n=-\infty}^{\infty} e^{in\pi x/L} \quad (5)$$

is the full periodic delta function.

The canonical momenta are computed according to, for example,

$$\pi_\phi = \frac{\partial \mathcal{L}}{\partial(\partial_+ \phi)}. \quad (6)$$

We find

$$\pi_\varphi = \partial_- \varphi - gA_n^+, \quad (7)$$

$$\pi_{\phi_0} = -gA_0^+, \quad (8)$$

$$\pi_{A_n^+} = \partial_+ A_n^+ - \partial_- A_n^- \equiv \pi_n^-, \quad (9)$$

$$\pi_{A_0^+} = \partial_+ A_0^+ \equiv \pi_0^-, \quad (10)$$

$$\pi_{A^-} = 0. \quad (11)$$

Together with the canonical Hamiltonian,

$$P^- = \int_{-L}^L dx^- \left[\frac{1}{2}(\pi^-)^2 - A^- (\partial_- \pi_n^- + g\partial_- \varphi) \right], \quad (12)$$

this represents a system with both first and second class constraints in the sense of Dirac [9]. To determine the appropriate quantum commutators we must first introduce gauge conditions and then either pursue the Dirac–Bergmann program [9], or seek to implement the equations of motion correctly as Heisenberg

equations. We shall consider two different gauges: the light-cone Weyl (or temporal) gauge, $A^- = 0$, and the light-cone Coulomb gauge, $\partial_- A^+ = 0$. It is easy to check that both gauges are consistent with the periodic boundary conditions we have imposed in the present light-cone formulation.

Details of the Dirac–Bergmann procedure have been given in many places, including in the light-cone literature [10,11], and we shall not repeat them here. We shall instead simply give the final field algebra and dynamical operators in each of the two gauges. In each case we then discuss the origin of the θ structure, and compute the chiral condensate.

3. Light-cone Weyl gauge

Imposing the condition $A^- = 0$, we find that the appropriate commutation relations are

$$[\pi_n^-(x^-), \partial_- \pi_n^-(y^-)] = \frac{ig^2}{2} \tilde{\delta}(x^- - y^-), \quad (13)$$

$$[A_n^+(x^-), \pi_n^-(y^-)] = i\tilde{\delta}(x^- - y^-), \quad (14)$$

$$[\partial_- \pi_n^-(x^-), \varphi(y^-)] = \frac{ig}{2} \tilde{\delta}(x^- - y^-), \quad (15)$$

$$[\varphi(x^-), \partial_- \varphi(y^-)] = \frac{i}{2} \tilde{\delta}(x^- - y^-), \quad (16)$$

$$[gA_0^+, \phi_0] = \frac{i}{2L}. \quad (17)$$

To these should be added the condition $\pi_0^- = 0$, which arises as a secondary constraint, and the Gauss law condition defining physical states,

$$G|\text{phys}\rangle = 0, \quad G \equiv (\partial_- \pi_n^- + g\partial_- \varphi), \quad (18)$$

which arises from the first-class constraint that remains after imposing the condition $A^- = 0$. The Hamiltonian is simply

$$P^- = \frac{1}{2} \int_{-L}^L dx^- (\pi_n^-)^2, \quad (19)$$

which does not immediately reflect that the physical spectrum of the theory is that of a free boson of mass g . This is evident only after one has satisfactorily implemented Eq. (18) and identified the physical subspace. That the correct spectrum is obtained for phys-

ical states can be seen, for example, by rewriting Eq. (19) as

$$P^- = \frac{1}{2} \int_{-L}^L dx^- \left(g\varphi - \frac{1}{\partial_-} G \right)^2. \quad (20)$$

Thus the correct physical spectrum is obtained in matrix elements between physical states,

$$\langle \text{phys} | P^- | \text{phys}' \rangle.$$

As usual, the Gauss operator, G , is the generator of residual, i.e., x^+ -independent, gauge transformations. A finite gauge transformation of this type is implemented by the unitary operator

$$\hat{U}[\omega] = \exp \left(i \int_{-L}^L dx^- \omega(x^-) G(x^-) \right), \quad (21)$$

with

$$\hat{U}[\omega] A^+ \hat{U}^\dagger[\omega] = A^+ + \partial_- \omega. \quad (22)$$

For periodic gauge functions ω , physical states satisfy

$$\hat{U}[\omega] |\text{phys}\rangle = |\text{phys}\rangle, \quad (23)$$

that is, they are invariant with respect to these residual gauge transformations. There exist gauge transformations, however, that are not themselves periodic yet still preserve the periodic boundary condition on the gauge field. These “large” gauge transformations may be decomposed into a product of a small transformation (21) and a transformation of the form

$$U_n = e^{in\pi x^-/L}, \quad (24)$$

where n is any integer. This specific structure is a consequence of the form of the symmetries of the original fermionic theory. The transformation (24) merely shifts the zero mode A_0^+ :

$$\delta A_0^+ = \frac{1}{ie} U_n \partial_- U_n^* = -\frac{n\pi}{eL}. \quad (25)$$

It will prove convenient to introduce the dimensionless field

$$z = \frac{eA_0^+ L}{\pi}, \quad (26)$$

in terms of which Eq. (24) takes

$$z \rightarrow z - n. \quad (27)$$

The Gauss law condition, Eq. (18), does not require the equivalence of physical states related by such a gauge transformation.

The situation is precisely analogous to that of equal-time quantization in the temporal gauge $A^0 = 0$. Physical states are invariant under the residual transformations obtained by exponentiating the Gauß operator, but only phase-invariant under the large gauge transformations analogous to (24). That is, if \hat{U}_n is the unitary operator that implements the transformation defined by (24), then

$$\hat{U}_n |\text{phys}\rangle = e^{-in\theta} |\text{phys}\rangle. \quad (28)$$

The specific form of the phase factor follows from the need to respect the composition law $\hat{U}_n \hat{U}_m = \hat{U}_{n+m}$.

In order to discuss the chiral condensate let us give a specific realization of the θ states. Since z commutes with the other fields, any state in the theory can be written as a superposition of states of the form

$$\psi_\theta(z) \otimes \Phi[A_n^+, \varphi]. \quad (29)$$

The state $\Phi[A_n^+, \varphi]$ can be thought of as either in a Fock or a functional Schrödinger representation, and must be annihilated by the Gauß operator in order to be in the physical subspace. The zero mode wavefunction $\psi_\theta(z)$ is chosen to be an eigenstate of \hat{U}_n with eigenvalue $e^{-in\theta}$. An explicit representation for \hat{U}_n is given by

$$\hat{U}_n = e^{-inp_z}, \quad (30)$$

where p_z is the momentum conjugate to the rescaled variable z :

$$p_z \equiv 2\sqrt{\pi}\phi_0, \quad (31)$$

so that

$$[z, p_z] = i. \quad (32)$$

That Eq. (30) is correct may be seen from

$$\hat{U}_n z \hat{U}_n^\dagger = z - n. \quad (33)$$

In this coordinate representation, p_z is represented as a derivative operator

$$p_z = -i \frac{d}{dz}, \quad (34)$$

and a convenient choice for the state $\psi_\theta(z)$ is

$$\psi_\theta(z) = e^{i\theta z}. \quad (35)$$

Note that this state is not, strictly speaking, normalizable. It does, however, satisfy the usual orthogonality relation

$$\langle \theta | \theta' \rangle = \int_{-\infty}^{\infty} dz \psi_\theta^*(z) \psi_{\theta'}(z) = \delta(\theta - \theta'). \quad (36)$$

With these explicit expressions for the states and operators in hand, we can now compute the chiral condensate $\langle \theta | \bar{\psi} \psi | \theta \rangle$. The physical θ vacuum state will be a tensor product of $\psi_\theta(z)$ with the Fock vacuum for A_n^+ and φ . Using the correspondence formula, Eq. (2), and the fact that the normal mode part of the vacuum is the Fock vacuum, we find that only the scalar zero mode contributes:

$$\begin{aligned} \langle \theta | \bar{\psi} \psi | \theta \rangle &= K \int_{-\infty}^{\infty} dz \psi_\theta^*(z) : \cos(2\sqrt{\pi}\phi_0) : \psi_{\theta'}(z) \quad (37) \\ &= K \int_{-\infty}^{\infty} dz \psi_\theta^*(z) \left(\frac{e^{ip_z} + e^{-ip_z}}{2} \right) \psi_{\theta'}(z). \quad (38) \end{aligned}$$

Making use of Eq. (28), and dividing out the normalization factor, we obtain the standard result

$$\langle \theta | \bar{\psi} \psi | \theta \rangle = K \cos \theta. \quad (39)$$

Note that the only place p_z appears in the theory is in the operator that implements large gauge transformations. In particular, there is no contribution from the zero mode sector to the Hamiltonian. This is actually unique to the light cone. In the equal-time formulation the two sectors decouple in the Hamiltonian [6]. Nevertheless, the standard result follows: the occurrence of the θ vacuum has no effect on the spectrum or other physical properties of the theory. Note also that the physically distinct values of θ lie in the range $0 \leq \theta \leq 2\pi$, again in accordance with the standard results [6].

4. Light-cone Coulomb gauge

The gauge $\partial_- A^+ = 0$ is in many ways the most natural one for this problem in that there is no residual first-class (Gauss law) constraint to be imposed on the states. Rather, the latter can be used to eliminate A^- at the operator level. (The zero mode of A^- may be set to zero by a purely x^+ -dependent gauge transformation.) Moreover, the Hamiltonian is just that of a free massive boson,

$$P^- = \frac{g^2}{2} \int_{-L}^L dx^- \varphi^2, \quad (40)$$

so that the physical spectrum of the theory is manifest. The relevant commutators are

$$[\varphi(x^-), \partial_- \varphi(y^-)] = \frac{i}{2} \tilde{\delta}(x^- - y^-) \quad (41)$$

$$[gA_0^+, \phi_0] = \frac{i}{2L}; \quad (42)$$

all other variables are eliminated by constraints. These commutators were first derived in Ref. [11]. Once again, the scalar field zero mode is the momentum conjugate to the gauge zero mode.

The light-cone Coulomb gauge condition, however, does not completely fix the gauge, due to Eq. (25). The large gauge transformations connect different Gribov regions [12]. We can eliminate this remaining gauge freedom by restricting z to lie in a “fundamental modular domain”, for example $0 \leq z \leq 1$ (with the points $z = 0$ and $z = 1$ identified). This uses up all remaining freedom and completely fixes the gauge.

Again, a general state can be represented as a superposition of states of the form $\psi(z)|\text{Fock}\rangle$, where the Fock state is constructed from the modes of φ . Here, without loss of generality, the function $\psi(z)$ may be taken to be periodic on the fundamental domain. A convenient representation in terms of a complete orthonormal set is

$$\Psi_n(z) = \frac{1}{\sqrt{2L}} e^{2in\pi z}. \quad (43)$$

The momentum operator p_z is again represented as a derivative, but the representation is not unique; the most general realization of the commutator (32) takes the form

$$p_z = -i \frac{d}{dz} + \theta. \quad (44)$$

Note that θ can be shifted out of p_z and into the states by the transformation

$$\xi_n^\theta(z) = e^{i\theta z} \Psi_n(z). \quad (45)$$

This new state satisfies the boundary condition $\xi_n^\theta(0) = e^{i\theta} \xi_n^\theta(1)$, and the transformed momentum operator is simply

$$\pi_z = -i \frac{d}{dz}. \quad (46)$$

Because the zero modes decouple completely from the normal modes in P^- , the physical vacuum will be the product state

$$|\theta\rangle = \xi_n^\theta(z)|0\rangle, \quad (47)$$

with $|0\rangle$ the light-cone Fock vacuum of the scalar field and ξ_n^θ any one of the wavefunctions of Eq. (45).

The condensate is now easily evaluated in the same way as before. Again, only the scalar field zero mode contributes:

$$\langle \theta | \bar{\psi} \psi | \theta \rangle = K \int_0^1 dz \xi_n^\theta(z)^* : \cos(p_z) : \xi_n^\theta(z). \quad (48)$$

Expressing ϕ_0 in terms of π_z and expanding the cosine then gives

$$\langle \theta | \bar{\psi} \psi | \theta \rangle = K \cos(2n\pi + \theta) = K \cos \theta. \quad (49)$$

As before, the zero mode operators do not appear in the Hamiltonian so that the value of θ has no effect on the spectrum of the theory. In addition, only the values $0 \leq \theta \leq 2\pi$ are physically distinct, as expected.

5. Chiral transformations

In the bosonized theory the chiral current is given by

$$J_5^\mu = \frac{1}{\sqrt{\pi}} \partial^\mu \phi. \quad (50)$$

The correct anomaly relation for J_5^μ follows directly from the equation of motion for ϕ :

$$\partial_\mu J_5^\mu = \frac{1}{\sqrt{\pi}} \partial_\mu \partial^\mu \phi = -\frac{e}{\pi} \epsilon_{\mu\nu} F^{\mu\nu}, \quad (51)$$

so that this aspect of the model is automatic in the bosonized version. In the fermionic theory the anomaly is quite nontrivial, involving a range of subtleties regarding the definition of singular operator products.

A conserved axial current can be defined, however, and is given by

$$J_{5,\text{sym}}^\mu \equiv \frac{1}{\sqrt{\pi}} (\partial^\mu \phi + g \epsilon^{\mu\nu} A_\nu). \quad (52)$$

$\partial_\mu J_{5,\text{sym}}^\mu = 0$ reflects the invariance of the theory under shifts in ϕ . The associated symmetry charge is

$$Q_{5,\text{sym}} = \int_{-L}^L dx^- J_{5,\text{sym}}^+ = -2z, \quad (53)$$

In both of the gauges we have discussed, this charge generates transformations that change the value of θ :

$$e^{i\alpha Q_{5,\text{sym}}} |\theta\rangle = |\theta - 2\alpha\rangle. \quad (54)$$

This is the only effect of a chiral transformation in the theory [6].

6. Discussion

We have seen that in light-cone quantization the θ vacuum structure of the bosonized Schwinger model can be reproduced by a careful treatment of the zero momentum modes of the fields defined on a compact space. The precise manner of its manifestation is somewhat gauge-dependent, as is familiar from the equal-time formulation. In the LC temporal gauge, one works in an extended Hilbert space and the residual gauge freedom is removed by identifying only those states that are annihilated by the Gauß operator as physical. The θ structure enters because the Gauß condition does not enforce gauge-equivalence of states related by certain “large” gauge transformations. These states are only phase invariant, with θ being the arbitrary phase that enters the transformation rule. This is precisely analogous to the equal-time formulation in the gauge $A^0 = 0$.

In contrast, the LC Coulomb gauge formulation is physical a one, in the sense that all gauge freedom can

be removed at the classical level and a purely physical Hilbert space employed. It is natural to do this and work in a finite “fundamental modular domain” for the gauge field zero mode. In this case, θ enters as an arbitrariness in the representation of the conjugate momentum p_z as a derivative, or, equivalently, as an arbitrariness in the boundary condition satisfied by the zero mode wavefunction on the fundamental domain. Again, this is quite familiar in the analogous equal-time context.

In either gauge the expected features of the model are reproduced, although these are rather simple. The spectrum of the theory in each case is that of a free boson of mass $g = e/\sqrt{\pi}$, and is independent of the value of θ . The only quantity that is sensitive to the value of θ is the chiral condensate, and its θ -dependence is correctly obtained. The crucial feature in each case is the presence of a vacuum wave function with the structure $\psi(z) \sim e^{i\theta z}$, along with the fact that the zero mode of the scalar field, which appears in the bosonized expression for $\bar{\psi}\psi$, is the momentum conjugate to the variable z . In the LC temporal gauge, the necessary vacuum wave function arises because states need only be phase-invariant under “large” (residual) gauge transformations. The role of the zero mode wave function is to supply this phase when acted on by the appropriate unitary operator. In the LC Coulomb gauge, a θ -dependent boundary condition on the fundamental domain is permissible, which leads to a similar structure in the zero mode wave function.

In the presence of a fermion mass, the θ vacuum has a definite impact on the spectrum of QED₁₊₁. That we obtain the correct results for the massless case, albeit in the bosonized form of the model, gives a reasonable basis for the extension to massive fermions.

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