2011

# Adding Value to Force Diagrams: Representing Relative Force Magnitudes

Paul J. Wendel
*Otterbein University*, pwendel@otterbein.edu

# Adding Value to Force Diagrams: Representing Relative Force Magnitudes

**Paul Wendel,** Mansfield University, Mansfield, PA

Nearly all physics instructors recognize the instructional value of force diagrams, and this journal has published several collections of exercises to improve student skill in this area.[1-4] Yet some instructors worry that too few students perceive the conceptual and problem-solving utility of force diagrams,[4-6] and over recent years a rich variety of approaches has been proposed to add value to force diagrams. Suggestions include strategies for identifying candidate forces,[6,7] emphasizing the distinction between "contact" and "noncontact" forces,[5,8] and the use of computer-based tutorials.[9,10] Instructors have suggested a variety of conventions for constructing force diagrams, including approaches to arrow placement and orientation[2,11-13] and proposed notations for locating forces or marking action-reaction force pairs.[8,11,14,15]

In recent years, a particular value-adding practice has become widespread among practitioners, that is, asking students to explicitly identify the object *receiving* each force as well as the object *exerting* each force on the diagram.[4,8,15-17] For example, rather than simply representing the gravitational attraction of a person toward the Earth as a downward-pointing arrow labeled "$\mathbf{F}_g$," students learn to label such an arrow as "$\mathbf{F}_{g \text{ on the person by the Earth}}$" or in a shorthand such as "$\mathbf{F}_{g \text{ person/Earth}}$".[18] Advocates believe that this practice discourages common conceptual pitfalls and enhances student understanding of physical situations.

## Ranking forces

Curriculum developers[8,17,19] have written exercises in which students rank forces from least to greatest. However, these authors do not suggest ranking forces as a routine practice in force diagram construction. About 10 years ago, a class of AP® students and I were working on a force ranking problem. As we struggled to use arrow lengths to represent relative

force magnitudes, a student suggested that as an alternative we could add hash marks to our force diagram arrows to help us remember relative force magnitudes. The student was borrowing the idea from his secondary geometry class, where hash marks are routinely used to rank the relative lengths of sides of geometric figures. The students' classmates and I were taken with the idea, and we adopted the practice and began to refine conventions for using hash marks to rank forces. In succeeding years I continued the practice and eventually extended it to all of my secondary physics courses. More recently, I have introduced the practice to introductory courses at the university level. In each case, most students appear to adopt the practice with relative ease, possibly because of familiarity with the use of hash marks in geometry classes.

## Examples for nonaccelerating objects

Consider a finger pushing downward on a book as it rests on a table [Fig . 1(a)].[20] Most students are able to produce a force diagram for the book similar to Fig. 1(b). Whether the students are working in small groups around a common whiteboard (typical) or as an entire class (atypical), I will ask the students how they know that the total amount of upward force (represented by three hash marks) is equal to the total amount of downward force (represented by a total of three hash marks). Beginners often answer that the total upward force must be equal to the total downward force because the book is stationary, after which I redirect their attention to the book's *acceleration* instead of the book's *velocity*. Next, I will ask how they know that $\mathbf{F}_{g \text{ on book by Earth}} > \mathbf{F}_{N \text{ on book by finger}}$. After some discussion, the students usually agree that they don't know for sure, i.e., that $\mathbf{F}_{g \text{ on book by Earth}} < \mathbf{F}_{N \text{ on book by finger}}$ is also possible. Often I will also ask whether it is possible that $\mathbf{F}_{g \text{ on book by Earth}} = \mathbf{F}_{N \text{ on book by finger}}$. Finally, since introductory students tend to conflate weight
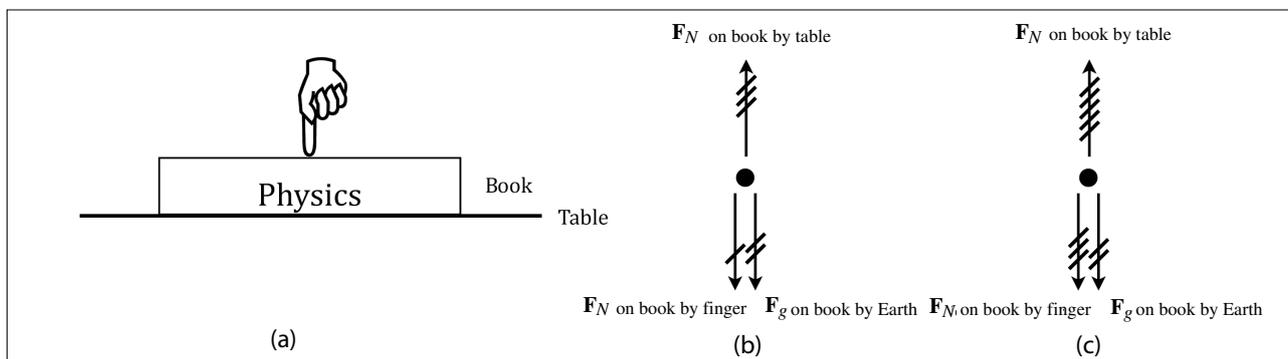


**Fig. 1. (a) A finger pushes downward on a book.[20] (b) A free-body diagram for the book while the finger pushes lightly. (c) A free-body diagram for the book while the finger pushes harder.**
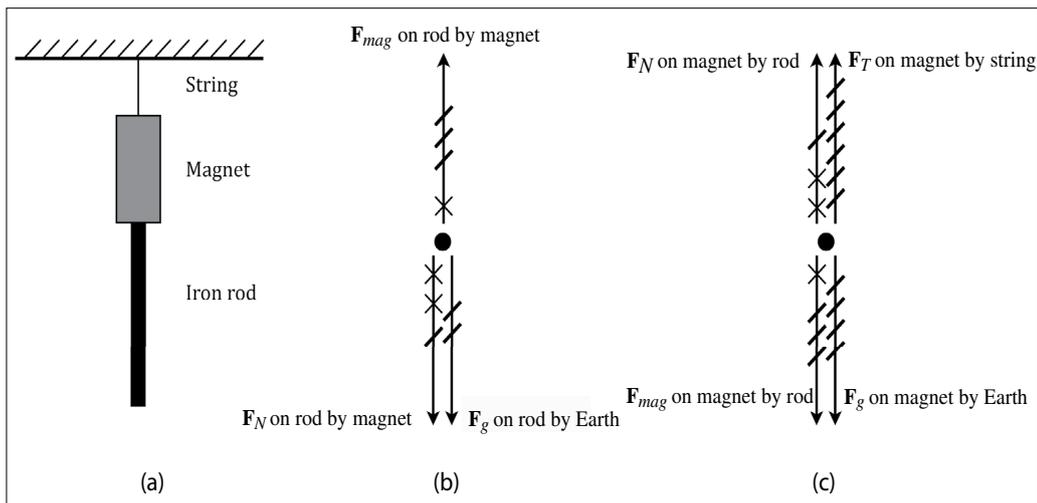
Fig. 2. (a) An iron rod is suspended from a magnet.[21] (b) A free-body diagram for the rod. (c) A corresponding free-body diagram for the magnet. Note that while the number of hash marks indicates relative magnitude, the number of X's distinguishes between different action-reaction pairs. The number of X's does not indicate magnitude.
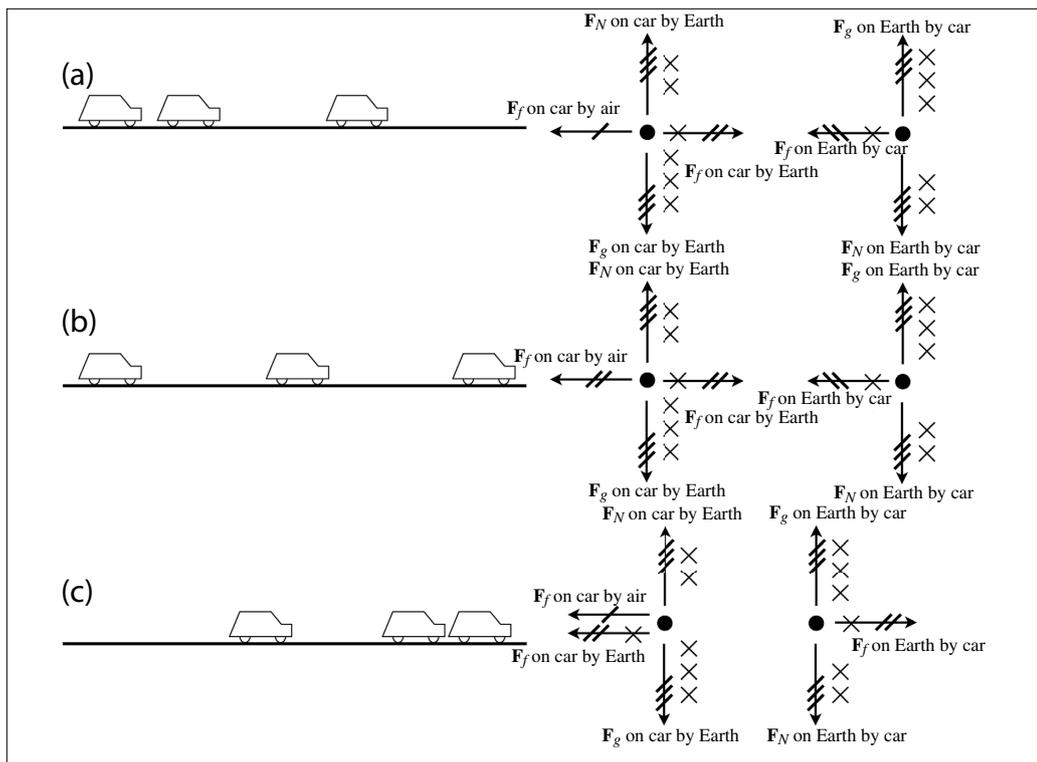


Fig. 3. (a) The driver steps on the gas pedal to bring the car to highway speed. (b) The driver steps on the gas pedal to maintain highway speed. (c) The driver steps on the brake to slow the car down. Force diagrams for the car and Earth are shown.

force with other forces, I will ask how the diagram will change if the finger pushes harder. Students typically add hash marks to $\mathbf{F}_g$, and in response I will ask how pushing harder on the book causes the Earth to pull harder on the book. Eventually they are able to produce something like Fig. 1(c). Throughout the discussion, the hash marks provide a shorthand that enables the students to focus on Newtonian concepts.

As a second example, consider an iron rod suspended by a magnet [Fig. 2(a)].[21] Following extensive discussion, most students are able to produce force diagrams similar to Figs. 2(b) and (c). Note that following McDermott & Shaffer's notation,[8] the X's and XX's mark action-reaction force pairs. (Different numbers of X's distinguish between different pairs and do *not* correspond to magnitudes of forces. Also note

that consistent with McDermott & Shaffer, no X's are added to forces whose reaction pair does not appear on the set of diagrams.) After the students come to agreement on the force diagrams, I will typically question them regarding how they know, e.g., that the total forces on each diagram are balanced or that $\mathbf{F}_{g \text{ on magnet by Earth}} > \mathbf{F}_{mag \text{ on magnet by rod}}$. Next, following McDermott & Shaffer I ask them to imagine that I can replace the magnet with a stronger magnet having the same weight. However, unlike McDermott & Shaffer I ask them to adjust the hash marks on their force diagrams. If necessary, I will use a different color marker to add two or three hash marks to $\mathbf{F}_{mag \text{ on rod by magnet}}$, then hand the marker to the students and ask them to rebalance the diagrams. A conceptually rich conversation normally ensues, including a discussion of

the nature of action-reaction force pairs and the students' surprise that increasing $\mathbf{F}_{mag}$ does not affect $\mathbf{F}_{g\text{ on magnet by Earth}}$, $\mathbf{F}_{g\text{ on rod by Earth}}$, or $\mathbf{F}_{T\text{ on magnet by string}}$.

In these and similar cases, hash marks provide a quick shorthand to determine whether or not the forces are balanced, enable quick modifications to force diagrams in follow-up questions, and encourage reliance on the force diagram as a thinking tool.

## Examples for accelerating objects

A driver steps on the gas to speed up a car along a straight, level highway [Fig. 3 (a)], maintains highway speed [Fig. 3(b)], and finally steps on the brake to slow the car down [Fig. 3(c)]. For simplicity, we ignore rolling friction in such problems. Adding hash marks to the force diagrams develops students' conceptual understanding of Newton's second law [forces on the car must be unbalanced in Fig. 3(a) and Fig. 3(c)], Newton's first law [forces on the car must be balanced in Fig. 3(b)], and Newton's third law (magnitudes of action-reaction force pairs must be equal). Student discussions of these force diagrams tend to be vigorous and productive. When the students have produced acceptable force diagrams, I typically ask how the diagrams change if the driver steps harder on the gas or brake pedal or how the diagrams change if we add a strong head wind. Particularly if students use a marker of a different color to modify the diagrams, the hash marks provide a convenient visual shorthand as the students discuss the resulting changes to the force diagrams.

Similarly rich and enlightening discussions of Newton's laws develop when students add hash marks to force diagrams for a person in an elevator who is ascending (or descending) at constant speed, speeding up, or slowing down. Many students are surprised to recognize that the person's weight ($\mathbf{F}_{g\text{ on person by Earth}}$) does not depend on the magnitude or direction of acceleration, and this is made visually clear as the number of hash marks on the person's weight remains unchanged from case to case.

## Evaluating student understanding

In my introductory physical science course this year (a freshman-level general education course for non-science majors), students solved no quantitative problems using Newton's laws. Instead, all practice, quiz, and exam questions on Newton's laws involved construction of force diagrams, including identification of objects receiving and exerting each force (referred to as on's and by's), identification of action-reaction force pairs, and use of hash marks to rank forces. Despite the nonquantitative nature of these exercises, students report that these exercises are challenging and instructive.

In my algebra-based general physics course, I require students to draw force diagrams. Part of an exam problem is shown in Fig. 4.[22] Of the 39 students who took the exam, 18 (46%) produced complete and correct force diagrams for both books, and, of these, 14 correctly calculated all of the forces on each diagram. By contrast, 21 students (54%) made at least one error in the force diagrams, and none of these students correctly calculated all of the forces. This is comparable to Rosengrant, Van Heuvelen, and Etkina's finding that when force diagrams were voluntary, students who drew correct force diagrams were more likely to answer questions correctly, while students who drew incorrect force diagrams were less likely to answer the question correctly compared to students who chose to draw no force diagram.[6]

## Adding value to force diagrams

From an instructor's point of view, hash marks add value to force diagrams by encouraging students to perform more of their analysis on the diagram. That is, instead of regarding a force diagram as a required step to be ignored upon completion, students are encouraged to use the force diagram as a thinking/analysis tool. To further encourage reliance on force diagrams, I also require students to record their numerical calculations on their force diagrams. Yet since I require the use of force diagrams, it is an open question whether the use



Fig. 4. The problem reads: "A person pushes two books across the table at a *constant speed* of 0.750 m/s as shown. The thesaurus mass is 2.04 kg, and the dictionary mass is 3.06 kg. There is sliding (kinetic) friction between the books and the table, but air friction is negligible. First, draw a force diagram for each book. Include on's, by's, and hash marks, and use X's to identify action-reaction force pairs. Next, the coefficient of (kinetic) friction is 0.250. Find each force on your force diagrams.
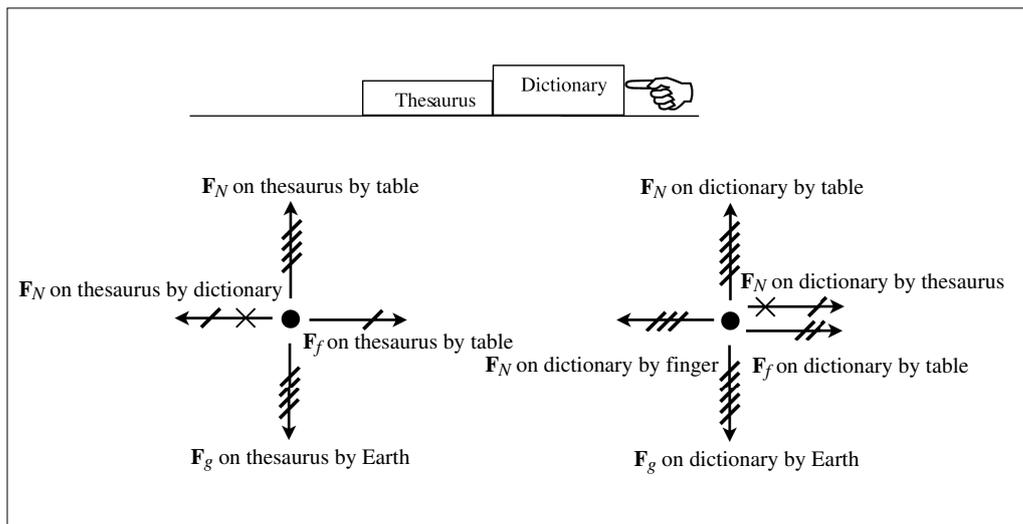
of hash marks adds value from the students' point of view. An informative study would compare voluntary use of force diagrams with hash marks to voluntary use of force diagrams without hash marks.
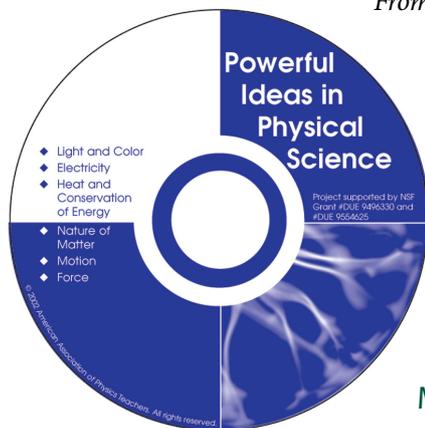
## References

1. J. E. Court, "Free-body diagrams," *Phys. Teach.* **31**, 104–108 (Feb. 1993).
2. J. E. Court, "Free-body diagrams revisited—I," *Phys. Teach.* **37**, 427–433 (Oct. 1999).
3. J. E. Court, "Free-body diagrams revisited—II," *Phys. Teach.* **37**, 490–495 (Nov. 1999).
4. K. Fisher, "Exercises in drawing and utilizing free-body diagrams," *Phys. Teach.* **37**, 434–435 (Oct. 1999).
5. R. Newburgh, "Force diagrams: How? and why?" *Phys. Teach.* **32**, 352 (Sept. 1994).
6. D. Rosengrant, A. Van Heuvelen, and E. Etkina, "Free-body diagrams: Necessary or sufficient?" Paper presented at the Physics Education Research Conference, Sacramento, CA (1994). From web.ebscohost.com/ehost/pdf?vid=3&hid=11&sid=a0ea9afae6 4f- 4c42-9890-0dd1a83a2f35%40sessionmgr13.
7. A. Van Heuvelen, "Overview, case study physics," *Am. J. Phys.* **59**, 898-907 (Oct. 1991).
8. L. C. McDermott, P. S. Shaffer, and the University of Washington Physics Education Group, *Tutorials in Introductory Physics* (Prentice Hall, Upper Saddle River, NJ, 2002).
9. S. Bonham, "Graphical response exercises for teaching physics," *Phys. Teach.* **45**, 482–486 (Nov. 2007).
10. R. J. Roselli, L. Howard, and S. Brophy, "A computer-based free body diagram assistant," *Comput. Appl. Engin. Ed.* **14**, 281–290 (2006).
11. B. Lane, "Why can't physicists draw FBD's?" *Phys. Teach.* **31**, 216–217 (April 1993).
12. W. Sperry, "Placing the forces on free-body diagrams," *Phys. Teach.* **32**, 353 (Sept. 1994).
13. M. Mattson, "Getting students to provide direction when drawing free-body diagrams," *Phys. Teach.* **42**, 398–399 (Oct. 2004).
14. A. Puri, "The art of free-body diagrams," *Phys. Ed.* **31**, 155–157 (1996).
15. E. van den Berg and C. van Huis, "Drawing forces," *Phys. Teach.* **36**, 222–223 (April 1998).
16. A. Arons, *Teaching Introductory Physics*, 3rd ed. (Wiley, New York, 1997).
17. D. P. Maloney, "Forces as interactions," *Phys. Teach.* **28**, 386–390 (Sept. 1990).
18. This is is sometimes referred to as "object/agent" notation. An alternative is "agent/object" notation: $F_{g \text{ by the Earth on the person}}$" or "$F_{g \text{ Earth/Person}}$." To my knowledge, a consensus notation has not yet emerged.
19. T. L. O'Kuma, D. P. Maloney, and C. J. Hieggelke, *Ranking Tasks Exercises in Physics* (Prentice Hall, Upper Saddle River, NJ, 1999).
20. From Ref. 8, HW-31.
21. From Ref. 8, 30.
22. Based on Ref. 8, 31-32.

**Paul Wendel** *taught high school physics for 18 years and currently serves as an assistant professor of physics and science education at Mansfield University. At the conclusion of the 2010-2011 academic year, Dr. Wendel will join the Knowles Science Teaching Foundation as a Teacher Developer.*

**pwendel@mansfield.edu**