2015

Letters to Joel

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What is the nature of mathematics?

This question, for very different reasons, was swirling in the heads of me (David) and my brother (Joel) in March 2003.

Joel

I had just seen *A Beautiful Mind,* the Academy Award–winning film inspired by Sylvia Nasar's biography of the mathematician John Nash. Two lines in the film stopped me in my tracks—one in which Nash refers to an "elegant" mathematical solution and one in which he insists that mathematics is an art. I am a professional musician with no formal mathematics beyond high school. In fact, I hated math in school. When I went to a music conservatory for my bachelor's degree, I considered it a huge plus that the school did not even offer a math class. Algebra, geometry, probability and statistics—these were among the most insufferably boring classes I ever had to pass. I knew some fellow students who liked math, of course, but I couldn't imagine why. Mathematics was a mindless, necessary, utilitarian task, like a frustrating menial job that makes you dread each impending shift. So Nash's assertions sounded ridiculous to me. But my interest was piqued.

David

I was in the final weeks of preparation of a new course—The History and Philosophy of Mathematics—that would serve both as a capstone for math majors at Otterbein University and as a state-mandated component for secondary-teaching licensure in mathematics. For me, this question was largely pedagogical: How do I, in a single term, convey to students the organic manner in which mathematics is always situated in the fabric of culture?

Joel, left, and David Stucki.

One of the main goals for students in this class is to learn to appreciate the evolving nature of mathematics and the way it has both influenced and been molded by culture.

As brothers pursuing different interests and vocations, with more than a decade separating us in years, each of us equally ignorant of the other's discipline, we nevertheless have in common a renaissance curiosity of the world and our relation to it. The events of that March triggered a dialogue that not only has persisted for more than 10 years, but that has left a permanent mark on Otterbein's curriculum.

The day after viewing the movie, Joel sent me the following email.

sigh…this is the third time I've had to write this today. stupid computers.

OK. I saw "A Beautiful Mind" last night, and was especially struck by two things. First, Ron Howard said that in speaking to the real John Nash and other mathematicians, he found out that they don't think in numbers so much as in shapes and relationships. Can you explain that?

The second question is the more philosophical one, and I just about drove myself insane this morning trying to wrap my mind around it. Nash's character describes one student's solution to a problem as "elegant," and also calls math an art. I have always thought of mathematics as an objective, exact science, and so I fail to see how it can be described in subjective terms. Math is used extensively IN art, music, architecture, etc., but this of course is a mere application. How can I conceive of it as an art in itself? I realize of course this means I need to define art. This is the part where my brain explodes.

I began to say that art was the indefinable expressive quality of something or other, but of course the word "indefinable" renders the rest of the definition moot. Then I said it was the original individualistic expression of an intellectual and/or technical pursuit. I liked that at first, then I realized it begs the question: what is expression? How can a human being, who judges everything by his own experiences, ever conceive of something totally original? I'm ultimately unable to define art, and I end up saying that anything subjective or with the possibility of variance can be artistic. My question, then, is this: "What is the nature of subjectivity in mathematics?"

I've probably just asked you to outline the entire body of mathematical study for the last few millennia. Sorry. Even a cursory understanding would be better than nothing. See, this is what I love about learning: interpretation; subjectivity. Put me in an algebra class and I'm asleep in five minutes. You learn some formula, and then you plug in any numbers and you get such-and-such an answer. No real thinking involved. The formula figures it out for you. But an equation, formula, pattern, etc., that can be INTERPRETED in different ways, that allows for creativity—that's interesting. I have no background in math, and so I don't know if I'm making sense. I hope I am.

Take your time.

Daniel Garrow

When I read this email I had two reactions: 1) Wow, what a cool conversation this will be; and 2) Hey, this would make an amazing writing assignment for the students in my course!

So, for the last decade I have given my students a copy of Joel's email and asked them to compose an email response that reflects on the notions of elegance and subjectivity in mathematics. The assignment, given in the first week of the term, both signals an expectation of new ways of thinking about and understanding mathematics, and serves as a bootstrap introduction to philosophical inquiries that will be threaded through the course.

Moreover, I believe that Joel's misconceptions are not only prevalent in our high schools, but they are also often reinforced by the curricula and teachers. Because the majority of students taking my course are prospective teachers, this connects well with those learning objectives aimed at licensure. Some excerpts of students' essays appear on pages 28 and 29.

Once I have collected and graded their essays, I share the rest of my dialogue with Joel. (This email exchange can be found at *maa.org/mathhorizons/supplemental.htm*.) Often this leads to discussions of how important an awareness and understanding of the historical and cultural development of mathematics is to K-12 pedagogy.

It is critical for teachers of mathematics to convey not only that equations and formulas work, but why they work. This is best understood in the historical context of their development and discovery. To know not just the proper application and use of a mathematical result, but also the questions that originally motivated it, gives deeper understanding and meaning to the practitioner, reducing the probability of misapplication.

It is equally important to see mathematics as a creative enterprise, rather than a monolithic and static edifice of "given" knowledge. To this end, Joel's questions and insights are helpful to my students.

Joel

After I wrote that first email, I remembered something from my high school algebra class. My grades were, perhaps, not as high as they could have been. I did just enough homework to scrape a respectable B. I spent an equal amount of time, however, engaging in my own independent musings. I started to wonder if there was a better way to solve the problems. Could I do them faster or more simply? Sometimes I shared my formulas in class, only to find out that they would result in correct answers only some of the time. Now, as I think about those two lines of that movie, I wonder if that's why a mathematician is an artist! Just because we solve a certain type of problem with a certain formula doesn't mean that's the only way to solve it. Someone had to come up with that formula, and perhaps he or she had to sift through other possibilities to arrive at it.

I had unwittingly tried to do some actual math as a freshman in high school, in spite of the efforts of the school board to discourage it! Of course, I didn't succeed in replacing the quadratic formula. My math was, you might say, rather badly done. But the only way I found math class worthwhile at all was if I forced myself to think. To create. To be an artist. ■

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Dear Joel . . .

Student responses to Joel's question have remained a part of David's class. These have been edited for space.

I think that the answer lies more in the thinking behind great mathematics rather than in the actual answers themselves. While many math problems have more than one solution, most of the ones the average student will encounter do not. Where the subjectivity and beauty of mathematics lies is in the variety of ways that someone can arrive at one single answer. Just like the traditional artist has certain preferences in which style of painting is best, so each mathematician has opinions about which method of arriving at an answer is best. Especially in high-level mathematics, your reference to "elegant" solutions becomes very applicable. While there may be dozens of ways to solve one problem, some solutions are more concise and understandable than others. Training oneself as a "math artist" involves attempts to reach higher levels of elegance in solutions and proofs.

Erin Moriarty (2004)

Often the most intriguing part of an artist's work is the ease with which the seemingly unrelated parts come together to form the beautiful whole. With mathematics, the formula itself is not the main point of interest, but the approach and the process that lead up to the final formula. These are the aspects of mathematics that are elegant.

The beauty of math is that it is the art of finding relationships. Think about writing. Words can be put together in simple ways that still have meaning, but they are not poetry until they are written to express an emotion. Their eloquence comes in [the poets'] ability to put ideas into words. Similarly, you can string a series of numbers together and make as many equations as you want, but they are not elegant until they have a meaningful pattern behind them. Mathematicians do not think in numbers, but in the relationships and patterns that are waiting to be found. This in itself is a beautiful thing to many mathematicians. The fact that nature is full of so many intricate patterns and regularities is the driving force behind their desire to make sense of it all.

Julie Carter (2004)

My own feelings about math and logic are based, in part, on an appreciation for the concrete understanding that is achievable. I see a beauty in this, in how something utterly complex can result in something so definitive. Or in how something seemingly simple can result in the terribly (but beautifully) complex (I am thinking of cellular automata). As you see the applications in nature of such things, physics, leaf patterns, there is a sort of awe-inspiring beauty, certainly a very subjective appreciation.

Secondly, I have a sense of this from the perspective of the research mathematician with whom I share much of my life. I have heard him talk repeatedly of beautiful and ugly math, of elegant proofs. He had explained the desire to relegate the ugly "number-crunching" sort of mechanisms to technical lemmas in order to end up with the elegant proof. This sort of reinforces my notion that the goal is the beauty of simplicity for that which is, actually, incredibly complex.

Kim Keiser (2004)

Daniel Garrow

Mathematics is a large puzzle, except the pieces aren't always there. You might have to dig under the sofa to find them, but when they are found, a beautiful image can emerge. The joy doesn't come from doing the puzzle, which is just rudimentary steps toward a goal, but instead comes from finding the missing pieces to make a whole.

Mathematics is music of the mind. Where music pleases the ears and art the eyes, mathematics brings pleasure to the brain. Although mathematics is typically thought of as a rigorous activity, the critical thought process can be very elegant. Like a brushstroke or pressing a key on a piano, mathematics in a small picture brings no beauty until you step back and view how everything meshes into an ultimate puzzle we are searching for the pieces of.

Shawn Winigman (2008)

Thinking of mathematics as numbers and variables is the equivalent of likening music to lines, clefs, notes, and rests! No sound! We, as in the collective society "we," have developed a style of teaching mathematics that severely handicaps our students' ability to develop their knowledge of mathematics. Our current approach to math education is like teaching a student how to play an instrument without allowing them [to] play it.

From a reductionist's perspective reducing anything to its most simple form is most certainly going to result in something of little worth. Reducing pictures to pixels, or music to vibrations takes all the value out of photography or painting. Likewise, with mathematics, reducing everything to numbers, variables, statements, and expressions has presented mathematics in a very unfair way.

Ultimately, in my mind, art is something that evokes feeling or emotion. Have you ever had the feeling where your hair stood on end and you got goose bumps all over? Maybe from a piece of music or a painting? Maybe it was just a musical chord. That's what art is. Some selections of music do nothing for me. This is I'm sure how you feel about much of mathematics, but sometimes after proving a theorem or forming a connection between two ideas I get goose bumps, and to me that is a resemblance that I'm "feeling" something. Hence, subjectively speaking, and based on my definition, mathematics is art.

Travis Kendall-Sperry (2011)

http://dx.doi.org/10.4169/mathhorizons.22.3.26